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The following Mathematics Florida Standards will be covered in this section:

- MAFS.912.N-RN.2.3: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; adding, subtracting, and multiplying polynomials.
- MAFS.912.A-SSE.1.1: Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
- MAFS.912.A-APR.1.1: Use the structure of an expression to identify ways to rewrite it.
- MAFS.912.A-APR.1.2: Rewrite expressions involving radicals and rational exponents using the properties of exponents.
- MAFS.912.N-RN.1.1: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents, allowing for a notation for radicals in terms of rational exponents.
- MAFS.912.N-RN.1.2: Rewrite expressions involving radicals and rational exponents using the properties of exponents.

**Topics in this Section:**

1. Using Expressions to Represent Real-World Situations
2. Understanding Polynomial Expressions
3. Algebraic Expressions Using the Distributive Property
4. Algebraic Expressions Using the Commutative and Associative Properties
5. Properties of Exponents
6. Operations with Radicals and Rational Exponents
7. Identifying Equivalent Expressions That Represent a Rational or Irrational Number
8. Understanding and Representing Radicals Form a Field
9. Operations with Rational and Irrational Numbers

The topics in this section will help students understand how to use expressions to solve real-world problems, identify the properties of polynomial expressions, and apply the distributive, commutative, and associative properties to simplify expressions.
Section 1: Expressions

### Topic 1

#### Using Expressions to Represent Real-World Situations

**Jenny** tweets 33 times a day. **Antonio** posts five tweets every day.

Let's represent the given number of days with a variable, $$d$$.

**Jenny's Total Tweets:**

Let's use an algebraic expression to describe Jenny's total posts after any given number of days.

**Creating an Algebraic Expression for Jenny's Total Tweets:**

- Jenny tweets 33 times a day. 
- Let $$d$$ represent the given number of days.

Thus, Jenny's total tweets after $$d$$ days can be expressed as:

$$33d$$

**Antonio's Total Tweets:**

Let's use an algebraic expression to describe Antonio's total posts after any given number of days.

**Creating an Algebraic Expression for Antonio's Total Tweets:**

- Antonio posts five tweets every day.
- Let $$d$$ represent the given number of days.

Thus, Antonio's total tweets after $$d$$ days can be expressed as:

$$5d$$

**Combined Total Posts:**

Let's use an algebraic expression to describe the combined total posts for Jenny and Antonio after any given number of days.

**Creating an Algebraic Expression for Combined Total Posts:**

- Jenny's total tweets: $$33d$$
- Antonio's total tweets: $$5d$$
- Combined total posts: $$33d + 5d = 38d$$

**After five days, how many tweets have Antonio and Jenny posted altogether?**

- Jenny's tweets after 5 days: $$33 \times 5 = 165$$
- Antonio's tweets after 5 days: $$5 \times 5 = 25$$
- Combined total after 5 days: $$165 + 25 = 190$$

**Let's Practice!**

1. **Mario and Luigi plan to buy a Wii U™ for their friends. Each controller costs $$29.99$$. They plan to purchase one console.**

   **a.** Use an algebraic expression to describe how much they will spend before sales tax based on purchasing the console and the number of games.

   **b.** If they purchase one console and three games, how much do they spend before sales tax? (Assume each game costs $$59.99$$).

   **c.** Mario and Luigi want to purchase some extra controllers for their friends. Each controller costs $$29.99$$. Use an algebraic expression to describe how much they will spend before sales tax based on purchasing the console, the number of games, and the number of extra controllers.

   **Let's Practice:**

   After five days, how many tweets have Antonio and Jenny posted altogether?
Section 1: Expressions

3. What is the total cost, before sales tax, if Mario and Luigi purchase one console, three games, and two extra controllers?

Try It!

2. Micah and Crystal purchase two movie tickets. Tickets cost $8.50 each. Drinks cost $3.50 each, and boxes of candy cost $3.00 each. Use an algebraic expression to describe how much they spend based on the number of drinks and boxes of candy they buy. Identify the parts of the expression by underlining the coefficient(s), circling the constant(s), and drawing a box around the variable(s).

When defining variables, choose variables that make sense to you, such as $h$ for hours.

BEAT THE TEST!

1. José is going to have the exterior of his home painted. He will choose between Krystal Klean Painting and Elegance Home Painting. Krystal Klean Painting charges $30.00 for 2 hours, Krystal Klean Painting charges $14.00 per hour. Elegance Home Painting charges $32.00 per hour. Let $h$ represent the number of hours for which José hires a painter. Which of the following statements are true? Select all that apply.

- The expression $30 + 14h$ represents the total charge for Krystal Klean Painting.
- The expression $32h$ represents the total amount José will spend on the painter's services.
- The expression $32h$ represents the total amount José will spend on the painter's services.
- If José hires the painters for 4 hours, Elegance Home Painting would be cheaper.
- If José hires the painters for 6 hours, Krystal Klean Painting would be cheaper.

Tip: When defining variables, choose variables that make sense to you, such as $x$ for dollars.
The Griffin family wants to buy an Xbox One + Kinect Sensor for $399. They also want to buy accessories and games. The wireless controllers cost $59.99 each. The headsets cost $99.99 each. The games cost $59.99 each.

Peter and Lois are trying to decide how many accessories and games to buy for their family. Let \( x \) represent the number of wireless controllers, \( y \) represent the number of headsets, and \( z \) represent the number of games the Griffins will purchase. Which of the following algebraic expressions can be used to describe how much the Griffins will spend, before sales tax, based on the number of accessories and games they purchase?

A. \( 3x + 2y + 2z \)  
B. \( 3x + y + 2z \)  
C. \( 4x + 2y + 2z \)  
D. \( 4x + 2y + z \)

This is an example of a polynomial expression. A polynomial can be one term or the sum of several terms. There are many different types of polynomials. A term is a constant, variable, or multiplicative combination of the two.

A monarch has one leader. How many terms do you think a monarch has? A bicycle has two wheels. How many terms do you think a bicycle has? A triceratops has three horns. How many terms do you think a triceratops has?
Section 1: Expressions

Let's recap:

<table>
<thead>
<tr>
<th>Type of Polynomial</th>
<th>Number of Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monomial</td>
<td>1</td>
</tr>
<tr>
<td>Binomial</td>
<td>2</td>
</tr>
<tr>
<td>Trinomial</td>
<td>3</td>
</tr>
<tr>
<td>Polynomial</td>
<td>More than 3</td>
</tr>
</tbody>
</table>

Some important facts:

- The degree of a monomial is the sum of the ___________ of the variables.
- The degree of a polynomial is the degree of the monomial term with the ___________ degree.
- Sometimes, you will be asked to write polynomials in standard form.
  - Write the monomial terms in ________________ _________ order.
  - The leading term of a polynomial is the term with the ___________
  - The leading coefficient is the coefficient of the ___________ _________.

Let's Practice!

1. Are the following expressions polynomials? If so, name the type of polynomial and state the degree. If not, justify your reasoning.

   a. $2m + 3n^{-1} + 8m^2$
   
   b. $6y^2 + 17ab^2 - 5a^2$
   
   c. $x^2 + 5x^3 + 9x^7$
   
   d. $10a^3 + 17ab^2 - 5a^2$
   
   e. $H + 3H - 5H$

2. If the type of polynomial and state the degree. If not, justify your reasoning.

   a. $\frac{3b}{2x^2}$
   
   b. $8x^2y^3$

Let's recap:
2. Are the following expressions polynomials?

a. \( x+y \)  
   - Polynomial
   - Not a polynomial

b. 3
   - Polynomial
   - Not a polynomial

c. \( x = y \)
   - Polynomial
   - Not a polynomial

d. \( x \cdot y \)  
   - Polynomial
   - Not a polynomial

e. \( x^2 + 3x - 4y^{-1} \)  
   - Polynomial
   - Not a polynomial

3. Consider the polynomial \( x^4 - 5x^3 + 9x^2 \).
   - What is the leading coefficient?
   - What is the leading term?
   - How many terms are in the polynomial?
   - What is the degree of the polynomial?
   - Write the polynomial in standard form.
### Section 1: Expressions

Now, use the distributive property to write an equivalent expression for \((a + 3)(a + 2)\).

<table>
<thead>
<tr>
<th>Column 1 (Left)</th>
<th>Column 2 (Right)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a + q)</td>
<td>(a + q)</td>
</tr>
<tr>
<td>(a + q)</td>
<td>(a + q)</td>
</tr>
<tr>
<td>(a + q)</td>
<td>(a + q)</td>
</tr>
</tbody>
</table>

1. Match the polynomial in the left column with its descriptive feature in the right column.

   - A. \(\frac{1}{2}\)
   - B. \(3^2\)
   - C. \(-H\)
   - D. \(0\)
   - E. \(1/2\)
   - F. \(3^3\)
   - G. \(-H\)
   - H. \(-H\)

II. Fifth degree polynomial
III. Seventh degree polynomial
IV. Leading coefficient of 3
V. Four terms
VI. Eighth degree polynomial
VII. Equivalent to \(3^2\)
VIII. Equivalent to \(4x^9 + 3x^8 - 9x^3\)

### Algebraic Expressions Using the Distributive Property

Recall the distributive property. One way to visualize the distributive property is to use models.

\[ (a + b)(c + d) = a(c + d) + b(c + d) \]

If \(a, b, c, d\) are real numbers, then
Section 1: Expressions

Let's Practice!

1. Write an equivalent expression for \(3x^2 + 2x - 6\) by modeling and then by using the distributive property.

2. Write an equivalent expression for \(x^2 - 3\) by modeling and then by using the distributive property.

3. Use the distributive property or modeling to write an equivalent expression for \(x + 5\) and \(x - 3\).

Try It!

1. Write an equivalent expression for \((x + 2)(x - 7)\).

2. Write an equivalent expression for \((x - 3)(x - 2)\) by modeling and then by using the distributive property.

3. Use the distributive property or modeling to write an equivalent expression for \((m + 5)(m - 3)\).
Section 1: Expressions

1. Students were asked to use the distributive property to write an equivalent expression for \( 5x - 2 \). Their work is shown below. Identify the student with the correct work. For the problems that are incorrect, explain where the students made mistakes.

**Student 1**

\( a \cdot q = p \cdot q \) and/or

\[ a \cdot b - 10 = p + q \]

The order of the numbers can be changed without affecting the order of subtraction. This is the **commutative** property.

Does it matter which number comes first?

What is \( 9 \cdot 2 \)?

What is \( 2 \cdot 9 \)?

Does it matter which number comes first?

What is \( 2 + 5 \)?

What is \( 5 + 2 \)?

**Student 2**

\( \frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b} \) and/or

\[ \frac{a}{b} - \frac{c}{d} = \frac{c}{d} - \frac{a}{b} \]

The order of the numbers can be changed without affecting the order of subtraction. This is the **commutative** property.

Does it matter which number comes first?

What is \( 9 \cdot 2 \)?

What is \( 2 \cdot 9 \)?

Does it matter which number comes first?

What is \( 2 + 5 \)?

What is \( 5 + 2 \)?

**Student 3**

\( 10 - 4x - 7x = \)

\[ 10 - x - 4x - 7x = \]

\( \frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b} \) and/or

\[ \frac{a}{b} - \frac{c}{d} = \frac{c}{d} - \frac{a}{b} \]

The order of the numbers can be changed without affecting the order of subtraction. This is the **commutative** property.

Does it matter which number comes first?

What is \( 9 \cdot 2 \)?

What is \( 2 \cdot 9 \)?

Does it matter which number comes first?

What is \( 2 + 5 \)?

What is \( 5 + 2 \)?
Let's look at some other operations and how they affect numbers.

Consider 3. What happens if you put parentheses around any two adjacent numbers? How does it change the product?

This is the **associative** property:

The grouping of the numbers does not change.

The **product** of the numbers does not change.

If \( a, b \) and \( c \) are real numbers, then

\[
(a + b) + c = a + (b + c)
\]

The associative property holds true for addition or multiplication. Does the associative property hold true for division or subtraction? If it does not, give a counterexample.

\[
\frac{(a + b)}{c} = \frac{a + (b + c)}{c}
\]

If \( a, b \), and \( c \) are real numbers, then

\[
\text{the distribution of the numbers can change and does not affect the division or multiplication.}
\]

Let's Practice!

Name the property (or properties) used to write the equivalent expression.

a. \((8 \cdot 4) \cdot 6 = 8 \cdot (4 \cdot 6)\)

b. \((8 \cdot 4) + 6 = (4 + 8) + 6\)

c. \((5 + (-3)) + 2 = 5 + ((-3) + 2)\)

1. Name the property (or properties) used to write the equivalent expression.

2. Let's look at some other operations and how they affect numbers.

subtile. The problem. The changes can be very subtle.

With properties, look closely at each piece of the problem.
Section I: Expressions

1. Try It!

2. Identify the property (or properties) used to find the equivalent expression.
   a. $M + H + N + H + M + H + N$

3. The following is a proof that shows $(3x)(z)$ is equivalent to $x y = (x)(z)$ used.
   Fill in each blank with either "commutative property" or "associative property" of addition.
   
   $M + x + N + y = = (x + y) + (M + N)$
   $x + y + (M + N) = (x + y) + (M + N)$

4. 4. $I \times (6 \times 13) = 4 \times (13 \times 6)$
   c. $\because (6 + 1) + 8 = 9 + (1 + 8)$
   q. $q \cdot (\alpha \cdot a) = (q \cdot \alpha) \cdot a$
   c. $(4 + 4) + 5 = (4 + 4) + 5$

5. Identify the property (or properties) used to find the equivalent expression.

   a. For the associative property of multiplication
   b. For the commutative property of addition
   c. For the commutative property of multiplication
   d. For the properties of real numbers to justify each step of the flow chart.

   1. Underline the differences in each step below. Then, use these abbreviations for the properties of real numbers to justify each step of the flow chart.
Let's review the properties of exponents.

What pattern do you notice?

Continuing the pattern, what would the following equal?

This is the zero exponent property: \( a^0 = 1 \).

Let's explore multiplying expressions with exponents and the same base.

This is the product property: \( a^m \cdot a^n = a^{m+n} \).

Let's explore dividing expressions with exponents and the same base.

This is the quotient property: \( \frac{a^m}{a^n} = a^{m-n} \).

Let's explore raising expressions with exponents to a power.

This is the power of a power property: \( (a^m)^n = a^{mn} \).

Let's explore multiplying expressions with exponents and the same base.
Section 1: Expressions

Let's explore raising products to a power.

\[(a \cdot b)^n = a^n \cdot b^n\]

This is the power of a product property:

\[\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right) = \frac{1}{8}\]

Let's explore raising quotients to a power.

\[\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\]

This is the power of a quotient property:

\[\left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2} = \frac{9}{25}\]

Let's Practice!

1. Determine if the following equations are true or false.
   Justify your answer.

   a. \(3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3\)
   b. \(5 \cdot 4^2 = 5 \cdot 16\)
   c. \(x^3 \cdot 4 = 4^3 \cdot x\)

   (Each equation is true.)

   \[\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}\]

   \[\left(\frac{4}{x}\right)^2 = \frac{4^2}{x^2}\]

   \[\left(\frac{z^2}{y}\right)^3 = \frac{z^6}{y^3}\]
Section 1: Expressions

1. Crosby and Adam were working with exponents. Try It!

Part A: Crosby claims that $\frac{3^2}{3^8}$. Adam argues that $3^2 \cdot 3^8$. Which one of them is correct? Use the properties of exponents to justify your answer.

Part B: Crosby claims that $\frac{3^2}{3^2}$. Adam argues that $2^6$. Which one of them is correct? Use the properties of exponents to justify your answer.

2. Use the properties of exponents to match each of the following with its equivalent expression.
Section 1: Expressions

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Section 1 – Topic 6
Radical Expressions and Expressions with Rational Exponents

Exponents are not always in the form of integers. Sometimes, you will see them expressed as rational exponents.

Use the properties of exponents to write the following expressions with rational exponents as expressions with integer exponents.

Do you notice a pattern? If so, what pattern did you notice?

Use this pattern and the properties of exponents to write the following expressions with rational exponents as radical expressions.

The rational exponent property:

Let's Practice!

1. Use the rational exponent property to write an equivalent expression for each of the following radical expressions.
   a. \( \sqrt[3]{8} \)
   b. \( \sqrt[4]{9} \)
   c. \( \sqrt[5]{125} \)

2. Use the rational exponent property to write each of the following expressions as integers.
   a. \( \sqrt[3]{8} \)
   b. \( \sqrt[4]{16} \)
   c. \( \sqrt[5]{25} \)
   d. \( \sqrt[6]{64} \)
   e. \( \sqrt[7]{343} \)
   f. \( \sqrt[8]{128} \)
Section 1: Expressions

1. Match each of the following expressions to its equivalent expression.

1. \( \sqrt[3]{m^3} \)
2. \( \sqrt[4]{4} \)
3. \( \sqrt[2]{1} \)
4. \( \sqrt[2]{\sqrt[3]{(m - 3)^3}} \)
5. \( \sqrt[3]{\sqrt[4]{(m - 3)^2}} \)
6. \( \sqrt[2]{\sqrt[2]{m^3}} \)

Try the Test!
Adding Expressions with Radicals and Rational Exponents

Let's explore operations with radical expressions and expressions with rational exponents.

To add radicals, the radicand of both radicals must be the same. To add expressions with rational exponents, the base and the exponent must be the same. In both cases, you simply add the coefficients.

Let's Practice!

1. Perform the following operations.
   a. \( \sqrt[3]{2} + 16 \)
   b. \( \sqrt[3]{3} + \sqrt[3]{3} \)
   c. \( \sqrt[4]{7} + \frac{7}{16} \)
   d. \( \sqrt[3]{5} + \sqrt[3]{5} \)
   e. \( \sqrt[3]{2} \cdot 3 \sqrt[3]{2} - 8 \cdot 3 \sqrt[3]{2} \)

2. \( \sqrt[5]{5} + \sqrt[5]{5} \)
   \( \sqrt[5]{5} + \sqrt[5]{5} \)

3. \( \sqrt[5]{2} + \sqrt[5]{2} \)

Let's Practice!
Section I: Expressions

Try It!

2. Perform the following operations.

a. $\sqrt{25} + 18^2 + 10^2$

b. $\sqrt{9} + 3 \cdot 6^2$

c. $\sqrt{50} + \sqrt{18} + \sqrt{10}$

d. $\sqrt{5} + 18^2 + 10^2$

e. $\sqrt{2} + \sqrt{8} + \sqrt{16}$

For radicals and expressions with rational exponents, always look for factors that are perfect squares when taking the square root (or perfect cubes when taking the cube root).
Section 1: Expressions

1. Which of the following expressions are equivalent to \( \frac{3}{2} \)?
   - \( \frac{3}{2} \)
   - \( \frac{3}{2} + \frac{3}{2} \)
   - \( \frac{3}{2} \cdot \frac{3}{2} \)
   - \( \frac{3}{2} \cdot \frac{3}{2} + \frac{3}{2} \)

2. Miguel completed the following proof to show that \( \frac{3}{2} \) is...
Section 1: Expressions

Let's Practice!

1. Use the properties of exponents to perform the following operations.

   a. \((\frac{x^2}{y^3}) \cdot (\frac{x^2}{y^3})\)

   b. \((\sqrt{y})^3\)

   c. \((\sqrt{x})^3\)

The properties of exponents also apply to expressions with rational exponents.

Let's explore multiplying and dividing expressions with radicals and rational exponents.

More Operations with Radicals and Rational Exponents

The properties of exponents also apply to expressions with rational exponents.
Section 1: Expressions

2. Use the properties of exponents to perform the following operations.

- a. \(M^3 \cdot N^2\)
- b. \(\frac{A^2}{B^3}\)
- c. \(3^3 \cdot 2^2\)
- d. \(\frac{A^3 \cdot B^2}{C^4}\)

Try it!
Section 1: Expressions

### Operations with Rational and Irrational Numbers

Let's review rational and irrational numbers.

- **Rational Numbers**: Numbers that can be represented as \( \frac{a}{b} \), where \( a \) and \( b \) are integers and \( b \neq 0 \). Numbers that can be represented in this form are examples of rational numbers.

- **Irrational Numbers**: Numbers that cannot be represented in this form are called irrational numbers. Radicals that are not perfect squares are examples of such numbers.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>25</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>223.666...</td>
</tr>
<tr>
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<td>0</td>
<td>( \frac{2}{3} )</td>
</tr>
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<td>0</td>
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<td>( \pi )</td>
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<tr>
<td>0</td>
<td>0</td>
<td>( \sqrt{8} )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>( \sqrt{9} )</td>
</tr>
</tbody>
</table>

### BEAT THE TEST!

1. Which of the following expressions are equivalent to \( \sqrt{2} \)?

   [ ] \( \frac{\sqrt{8}}{2} \)
   [ ] \( \frac{\sqrt{2}}{4} \)
   [ ] \( \sqrt{2} \)
   [ ] \( \sqrt{8} \)
   [ ] \( \sqrt{2} \times \sqrt{2} \)
Section 1: Expressions

Given two rational numbers \( a \) and \( b \), what can be said about the sum of \( a + b \)?

Given two irrational numbers \( a \) and \( b \), what can be said about the sum of \( a + b \)?

Given a non-zero rational number \( a \) and an irrational number \( b \), what can be said about the product of \( a \cdot b \)?

Given two irrational numbers \( a \) and \( b \), what can be said about the product of \( a \cdot b \)?

Given a rational number \( a \) and an irrational number \( b \), what can be said about the sum of \( a + b \)?

Given two irrational numbers \( a \) and \( b \), what can be said about the sum of \( a + b \)?

The product of \( a \) and \( b \)?

The sum of \( a \) and \( b \)?

The product of \( a \) and \( b \)?

The sum of \( a \) and \( b \)?
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Section 1: Expressions

1. Let $a$ and $b$ be non-zero rational numbers and $c$ and $d$ be irrational numbers. Consider the operations below and choose whether the result can be rational, irrational, or both.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a + b$</td>
<td>Rational</td>
</tr>
<tr>
<td>$a - b$</td>
<td>Rational</td>
</tr>
<tr>
<td>$a \cdot b$</td>
<td>Irrational</td>
</tr>
<tr>
<td>$a \div b$</td>
<td>Irrational</td>
</tr>
</tbody>
</table>

2. Consider the following expression.

$2 + \sqrt{3}$
## Section 2: Equations and Inequalities

### Mathematics Florida Standards

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAFS.912.A-SSE.1.2</td>
<td>Use the structure of an expression to identify ways to rewrite it.</td>
</tr>
<tr>
<td>MAFS.912.A-REI.1.1</td>
<td>Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution.</td>
</tr>
<tr>
<td>MAFS.912.A-REI.1.3</td>
<td>Solve simple equations and inequalities in one variable, including equations with all terms on one side and inequalities of the form</td>
</tr>
</tbody>
</table>

### Topics in This Section

1. **Topic 1: Equations: True or False**
   - Forming a coordinate plane, often forming a curve (which could be a line).
   - All its solutions plotted in the graph of an equation in two variables is the set of ordered pairs (x, y) that satisfy the equation.
2. **Topic 2: Identifying Properties When Solving Equations**
   - Identify properties of equality that allow the solutions of the equation to be rearranged.
3. **Topic 3: Solving Equations Using the Zero Product Property**
   - Use the structure of an expression to rewrite it.
4. **Topic 4: Solving Equations Using the Zero Product Property**
   - Identify properties of equality that allow the solutions of the equation to be rearranged.
5. **Topic 5: Solving Inequalities**
   - Use the structure of an expression to rewrite it.
6. **Topic 6: Solving Inequalities**
   - Use the structure of an expression to rewrite it.
7. **Topic 7: Solving Compound Inequalities**
   - Use the structure of an expression to rewrite it.
8. **Topic 8: Solving Absolute Value Equations and Inequalities**
   - Use the structure of an expression to rewrite it.
9. **Topic 9: Rearranging Formulas**
   - Use the structure of an expression to rewrite it.
10. **Topic 10: Solution Sets to Equations with Two Variables**
    - Use the structure of an expression to rewrite it.

### Topics Covered in This Section

- MAFS.912.A-SSE.1.2
- MAFS.912.A-REI.1.1
- MAFS.912.A-REI.1.3
- MAFS.912.A-SSE.1.2
- MAFS.912.A-REI.1.4
- MAFS.912.A-REI.2.3

- MAFS.912.A-REI.1.3
- MAFS.912.A-SSE.1.2
Section 2: Equations and Inequalities

Equations: True or False?

Consider the statement $4 + 5 = 2 + 7$. This is a grammatically correct sentence. Is the sentence true or false?

Consider the statement $1 + 3 = 8 + 6$. This statement is also a grammatically correct sentence. Is the sentence true or false?

The previous statements are examples of number sentences. A number sentence is a statement of equality between two expressions. If both numerical expressions are equal, the number sentence is said to be true. If both numerical expressions don't equal the same number, we say the number sentence is false. True and false statements are called truth values.

Let's Practice!

1. Determine whether the following number sentences are true or false. Justify your answer.
   a. $13 + 4 = 7 + 11$
   b. $\frac{2}{5} + \frac{8}{14} = 0.275$

Try It!

2. Determine whether the following number sentences are true or false. Justify your answer.
   a. $(83 \cdot 401 \cdot 638 + 401 \cdot 638 \cdot 83)$
   b. $(6 + 4) \cdot (3 + 2) = 3 + 2 \cdot (6 + 4)$

The previous statements are examples of number sentences. Which sentence is correct? Consider the statement $1 + 3 = 8 + 6$. This statement is also a grammatically correct sentence. Is the sentence true or false? Consider the statement $4 + 5 = 2 + 7$. This is a grammatically correct sentence. Is the sentence true or false?
Section 2: Equations and Inequalities

A number sentence is an example of an algebraic equation.

An algebraic equation is a statement of equality between two ____________________.

Algebraic equations can be number sentences (when both expressions contain only numbers), but often they contain __________________ whose values have not been determined.

Consider the algebraic equation $4(x + 2) = 4x + 8$.

What values could we substitute for $x$ to make it a true sentence?

Consider the algebraic equation $x + 6 = x + 9$.

What does this tell you about the numbers we can substitute for $x$?

Let's Practice!

3. Consider the algebraic equation $x + 3 = 9$.

a. What value can we substitute for $x$ to make it a true number sentence?

b. How many values could we substitute for $x$ and have a true number sentence?

Try It!

4. Consider the algebraic equation $x + 6 = x + 9$.

What values could we substitute for $x$ to make it a true sentence?

5. Complete the following.

a. $x^2 = 4$ is true for ____________________.

b. $2x + x = x$ is true for ____________________.

c. $x + 6 = x + 8$ is true for ____________________.
Section 2: Equations and Inequalities

Identifying Properties When Solving Equations

Section 2 - Topic 2

BEAT THE TEST!

1. Which of the following have the correct solution? Select all that apply.

2. $x + 5 = 12$

3. $2x = 8$

4. $x - 9 = 3$

This brings us to some more properties that we can use to write equivalent expressions.

The following equations are equivalent. Describe the operation that occurred in the second equation.

$3 + 5 = 8$ and $3 + 5 - 5 = 8 - 5$

$\frac{z}{2} = 4$ and $\frac{z}{2} = 4$

$2 + x + 3 = 7$ and $2 + x + 3 = 7$

$5 + 3 = 8$ and $5 + 3 - 3 = 8 - 3$

$8 = x + 5 + x = 10$ and $10 = x + 5 + x = 10$

$z = x - 2 + x + 3$ and $z = x - 2 + x + 3$

$2x + 5 = 19$ and $2x + 5 = 19$
Section 2: Equations and Inequalities

Properties of Equality

If $x$ is a solution to an equation, it will also be a solution to the new equation formed when the same number is added to or subtracted from each side of the original equation. These are the addition and subtraction properties of equality.

\[
\frac{2}{3} = \frac{2}{3} \quad \text{and} \quad \frac{2}{3} + \frac{1}{3} = \frac{2}{3} + \frac{1}{3}
\]

Give examples of this property.

Let's Practice!

1. The following equations are equivalent. Determine the property that was used to write the second equation.

   a. $x = 3x + 5$ and $x = 3x + 5$

   b. $x = 3x + 5$ and $x = 3x + 5$

   c. $x = 3x + 5$ and $x = 3x + 5$

These are the multiplication and division properties of equality.

If $x$ is a solution to an equation, it will also be a solution to the new equation formed when the same number is multiplied by or divided into each side of the original equation.

\[
\frac{a}{c} = \frac{a}{c} \quad \text{and} \quad \frac{a}{c} \cdot \frac{b}{d} = \frac{a}{c} \cdot \frac{b}{d}
\]

Give examples of this property.

1. The following equations are equivalent. Determine the property that was used to write the second equation.

   a. $x = 3x + 5$ and $x = 3x + 5$

   b. $x = 3x + 5$ and $x = 3x + 5$

   c. $x = 3x + 5$ and $x = 3x + 5$

Properties of Equality
### Section 2: Equations and Inequalities

#### Try It!

1. For each algebraic equation, select the property or properties that could be used to solve it.

<table>
<thead>
<tr>
<th>Commutative Property</th>
<th>Distributive Property</th>
<th>Multiplication or Division Property of Equality</th>
<th>Addition or Subtraction Property of Equality</th>
<th>Algebraic Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
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</tbody>
</table>

2. The following equations are equivalent. Determine the property that was used to write the second equation.

   a. \(8x = 6\) and \(\frac{8}{x} = \frac{8}{1}\) 
   
   b. \(8 + 8 = 16\) and \(8 + 8 = 16\) 
   
   c. \(2(x + 4) = 14 - 6x\) and \(2x = 14 \Rightarrow 6x + 6x = 14 - 6x\) 
   
   d. \(8x = 6\) and \(\frac{8}{x} = \frac{8}{1}\) 
   
   e. \(2(x + 8) = 14 - 6x\) and \(2x + 8 = 14 - 8\)
## Solving Equations

**Sometimes, you will be required to justify the steps to solve an equation.** The following equation is solved for $x$.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5(x + 3) - 2 = 2 - x + 9$</td>
<td>Given</td>
</tr>
<tr>
<td>$5x + 15 - 2 = 2 - x + 9$</td>
<td>p.</td>
</tr>
<tr>
<td>$5x + 13 = 11 - 13$</td>
<td>e.</td>
</tr>
<tr>
<td>$5x = -2$</td>
<td>f. Equivalent Equation</td>
</tr>
<tr>
<td>$x = \frac{-2}{5}$</td>
<td>g.</td>
</tr>
<tr>
<td>$\frac{9}{2} = x$</td>
<td>h.</td>
</tr>
<tr>
<td>$x = \frac{9}{2}$</td>
<td>i.</td>
</tr>
</tbody>
</table>

**Other times, you may be required to write and solve an equation for a situation.**

Consider the following scenario. Your class is raising funds for an end of the year trip to an amusement park. Your class plans to rent one bus. It costs $150.00 to rent a school bus for the day plus $33.00 per student admission ticket. Your class has raised $1,200 so far. Write an expression to represent the amount of money the school needs to raise. Write an equation to represent the number of students that will be able to attend the trip. How many students can attend the trip if your class raises an additional $1,800?
Section 2: Equations and Inequalities

Let's Practice!

1. Consider the following equation $2x - 3(2x - 1) = 3 + 12x - 5$.
   - Solve the equation for $x$.
   - For each step, identify the property used to write an equivalent equation.

Try It!

2. Consider the following equation $3(4 + x + 1) = 3 + 12x - 5$.
   - Solve the equation for $x$.
   - For each step, identify the property used to convert the equation.

Some equations, such as $2x = 2x$, have all real numbers as the solution. No matter what number we substitute for $x$, the equation would still be true.

3. A high school surveyed its student population about their favorite sports. The 487 students who listed soccer as their favorite sport represented 17 fewer students than three times the number of students who listed basketball as their favorite sport. Write and solve an equation to determine how many students listed basketball as their favorite sport.

Some equations, such as $2x + 5 = 2x + 1$, have no solution. There is no number we could substitute for $x$ that would make the equation true.

Let's Practice!

1. Consider the following equation $2x - 3(2x - 1) = 3 + 12x - 5$.
   - Solve the equation for $x$.
   - For each step, identify the property used to write an equivalent equation.

2. Consider the following equation $3(4 + x + 1) = 3 + 12x - 5$.
   - Solve the equation for $x$.
   - For each step, identify the property used to convert the equation.

Some equations, such as $2x = 2x$, have all real numbers as the solution. No matter what number we substitute for $x$, the equation would still be true.
Section 2: Equations and Inequalities

BEAT THE TEST!

1. The following equation is solved for \( x \).

   Use the properties to justify the reasons for each step in the chart below.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 2(x - 3) + (x + 5) = 0 )</td>
<td>Given</td>
</tr>
<tr>
<td>( 2x - 6 + x + 5 = 0 )</td>
<td>Equivalent Equation</td>
</tr>
<tr>
<td>( 3x - 1 = 0 )</td>
<td>Equivalent Equation</td>
</tr>
<tr>
<td>( 3x = 1 )</td>
<td>Equivalent Equation</td>
</tr>
<tr>
<td>( x = \frac{1}{3} )</td>
<td>Equivalent Equation</td>
</tr>
</tbody>
</table>

This is the zero product property.

If someone told you that the product of two numbers is zero, what could you say about the two numbers?

If someone told you that the product of two numbers is 1, what could you say about the two numbers?

This is the zero product property.

Solving Equations Using the Zero Product Property

Section 2 – Topic 4

Describe how to use the zero product property to solve the equation: \( (x - 3)(x + 9) = 0 \). Then, identify the solutions.

\( a \) = 0, \( b \) = 0, \( c \) = 0, \( d \) = 0. If \( ab = 0 \), then either \( a = 0 \) or \( b = 0 \). If someone told you that the product of two numbers is zero or one, what could you say about the two numbers?
Section 2: Equations and Inequalities

Let's Practice!

1. Identify the solution(s) to $2^y (y - 3) (6 - y) = 0$.

2. Identify the solution(s) to $(2x - 5)(x + 1) = 0$.

Try It!

3. Michael was given the equation $(x - 7) (x + 11) = 0$ and asked to find the zeros. His solution set was $\{-11, 7\}$. Explain whether you agree or disagree with Michael.

4. Identify the solution(s) to $(x - 3) (x + 11) = 0$. Let's Practice!
Section 2: Equations and Inequalities

BEAT THE TEST!

1. Use the values below to determine the solutions for each equation.

\[-2 \cdot 3 = \frac{4}{5} \cdot \frac{4}{3} \cdot 0 \cdot 6\]

\[0 = (z - 1)(z + 1)\]

\[0 = (3 - x)(2 + x)\]

\[0 = (6 - x)(3 + x)\]

\[0 = (4 - u)(2 - u)\]

\[0 = (4 + v)(1 + v)\]

\[\begin{array}{c|c|c|c|}
\hline
-2 & \frac{4}{5} & 0 & 6 \\
\hline
\frac{4}{3} & \frac{2}{1} & \frac{7}{2} & \frac{7}{2} \\
\hline
\frac{5}{4} & 3 & 2 & 0 \\
\hline
\end{array}\]

Let's start by reviewing how to graph inequalities.

Solving Inequalities - Part 1

Section 2 - Topic 5

When the endpoint is an _________ dot or circle, the
number represented by the endpoint
\hspace{1cm} part of the solution set.

Describe the numbers that are graphed in the example above.

When the endpoint is an _________ dot or circle, the
number represented by the endpoint
\hspace{1cm} part of the solution set.

Can you list all the numbers? Explain your answer.

Describe the numbers that are graphed in the example above.

Let's start by reviewing how to graph inequalities.
Consider the following graph.

When the endpoint is a _______ dot or circle, the number represented by the endpoint is a part of the solution set.

Write an inequality that represents the graph above.

Write the solution set that represents the graph above.

Why is "or equal to" included in the solution set?

Just like there are Properties of Equality, there are also Properties of Inequality. There are also Properties of Inequality.

Addition and Subtraction Property of Inequality:

If $a < b$, then $a + c < b + c$ and $a - c < b - c$ for any real number $c$.

Consider the inequality $(2x - 1) + 2 \leq 1 - 0.2$. Use the addition or subtraction property of inequality to solve for $x$.

Let's Practice!

1. Consider the inequality $(4 + x) - 5 \geq 1 - 0.2$. Use the addition or subtraction property of inequality to solve for $x$.

Express the solution in set notation and graphically on a number line.
Section 2: Equations and Inequalities

2. Consider the following inequality:
\[ 4 + x < 5 \]
Use the addition or subtraction property of inequality to solve for \( x \).
Express the solution in set notation and graphically on a number line.

3. Peter deposited $27 into his savings account, bringing the total to over $234. Write and solve an inequality to represent the amount of money in Peter’s account before the $27 deposit.

Consider \( x \) and \( 2x < 2 \cdot 5 \).
Identify a solution to the first inequality.
Show that this solution makes the second inequality true.

Consider \( x = 5 \) and \( -2x > -2 \cdot 5 \).
Identify a solution to the first inequality.
Show that this solution makes the second inequality false.

How can we change the second inequality so that the solution makes it true?

Consider \( -x = 5 \).
Use the addition and/or subtraction property of inequality to solve.
Section 2: Equations and Inequalities

Multiplication Property of Inequality

If \( a \equiv b \), then for any positive real number \( k \), \( ka \equiv kb \).

If \( a \equiv b \), then for any negative real number \( k \), \( ka \equiv kb \).

The same property is true when dealing with \( \geq \) or \( \leq \).

Let's Practice!

1. Find the solution set to each inequality. Express the solution in set notation and graphically on a number line.
   - a. \(-7y - 7 > 4\)
   - b. \(7y + 8 \geq 9\)

2. At 5:00 PM in Atlanta, Georgia, Ethan noticed the temperature outside was 72°F. The temperature decreased at a steady rate of 2°F per hour. At what time was the temperature less than 64°F?

   If \( a > b \), then for any negative real number \( k \), \( ak < bk \).

   If \( a > b \), then for any positive real number \( k \), \( ak > bk \).

   If \( a < b \), then for any negative real number \( k \), \( ak > bk \).

   If \( a < b \), then for any positive real number \( k \), \( ak < bk \).
Section 2: Equations and Inequalities

Try It!

3. Find the solution set to the inequality. Express the solution in set notation and graphically on a number line.

a. \(-6(x - 5) > 42\)

b. \(4(x + 3) \geq 2(2x - 2)\)

Part A: How many miles will Ulysses have to drive for Option A to be the cheaper option?

Part B: How many miles will Ulysses have to drive for Option B to be the cheaper option?

BEAT THE TEST!

1. Ulysses is spending his vacation in South Carolina. He rents a car and is offered two different payment options. He can either pay $10.00 each day plus $0.40 per mile (option A) or pay $25.00 each day plus $0.15 per mile (option B). Ulysses rents the car for one day.

Part A: Write an inequality representing the number of miles where option A will be the cheaper plan.

Part B: How many miles will Ulysses drive for Option A to be the cheaper option?
2. Stephanie has just been given a new job in the sales department of Frontier Electric Authority. She has two salary options. She can either receive a fixed salary of $50,000 per week or a salary of $20,000 per week plus a 5% commission on her weekly sales. The variable $w$ represents Stephanie's weekly sales.

Which solution set represents the dollar amount of sales that she must generate in a week in order for the option with commission to be the better choice?

A. $w \geq 300,000$
B. $w \geq 700,000$
C. $w \geq 3\times10^6$
D. $w \geq 6\times10^6$

Consider the following options:

Option A: You get to play Call of Duty after you clean your room or do the dishes.

Option B: You get to play Call of Duty after you clean your room or do the dishes.

What is the difference in Option A and B?

Consider the following equations and inequalities:

1. $15 - 20 < 0$ or $2.5 + 3.5 = 7$
2. $0 < 4 - 6$ or $3 + 2 = 6$
3. $0 < 4$ if $9 + 2 > 0$
4. $2 + 9 > 10$ and $10 > 6$

true

Consider the following statements that are true.

Solving Compound Inequalities
Section 2 – Topic 7
Section 2: Equations and Inequalities

These are called compound equations or inequalities.

When the two statements in the previous sentences were joined by the word AND, the compound equation or inequality is true only if _________ statements are true.

When the two statements in the previous sentences were joined by the word OR, the compound equation or inequality is true if at least _________ of the statements is true. Therefore, it is also considered true if _________ statements are true.

Let's graph $x \leq 6$ and $x \geq 1$.

We read this as “$x$ is greater than one ________ less than six.”

Many times this is written as $1 < x < 6$. This notation denotes the conjunction “and.”

How many solutions does this inequality have?

Many times this is written as $1 < x < 6$. This notation denotes the conjunction “and.”

We read this as “$x$ is greater than one ________ less than six.”

Let's Practice!

1. Consider $x > 1$ or $x < 6$. Could we write the inequalities above as $1 < x < 6$? Explain your answer.

2. Graph the solution set to each compound inequality on a number line.
   a. $x < 2$ or $x > 5$
   b. $x > 6$ or $x < 6$
   c. $1 < -x < 7$

These are called compound equations or inequalities.
Section 2: Equations and Inequalities

Try It!

4. Graph the solution set to each compound inequality on a number line.

a. \( x \leq 1 \) or \( x \geq 8 \)

b. \( x \geq 6 \) or \( x \leq 4 \)

c. \(-6 \leq x \leq 4\)

3. Write a compound inequality for the following graphs.

a. Compound inequality: \( x \geq 4 \) or \( x < 7 \)

b. Compound inequality: \( x \leq 9 \) or \( x > 2 \)

c. Compound inequality: \( x > 1 \) or \( x < 8 \)

Tip

When solving inequalities, you will need to reverse the inequality symbol when you multiply or divide by a negative value. Be on the lookout for negative coefficients.
Section 2: Equations and Inequalities

5. Write a compound inequality for the following graphs.

a. Compound inequality:

b. Compound inequality:

BEAT THE TEST!

1. Use the terms and symbols in the bank to write a compound inequality for each of the following graphs. You may only use each term once, but you do not have to use all of them.

<table>
<thead>
<tr>
<th>3x</th>
<th>≤</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>7x</td>
<td>≥</td>
<td>14</td>
</tr>
</tbody>
</table>

-6 ≤ x ≤ 17
15 < x
Section 2: Equations and Inequalities

Consider \( |a| < 5 \).

Using our definition of absolute value, this is saying that \( a \) is \( \frac{5}{5} \) units away from zero on the number line.

What are some numbers that could be represented by \( a \) on a number line?

Graph all the numbers represented by \( a \) on a number line.

For any real numbers \( a \) and \( b \), if \( |a| = |b| \), then \( a = b \) or \( b = a \).

This is the absolute value of a number.

For example, \( |f| = 5 \), so \( f = \frac{5}{5} \) or \( f = \frac{-5}{5} \).

This is written as \( |f| = 5 \).

How far away is "\( -9 \)" from zero on the number line?

This is written as \( |-9| \).

How far away is "\( 9 \)" from zero on the number line?

This is written as \( |9| \).

Consider \( |c| > 5 \).

Solving Absolute Value Equations and Inequalities
Consider \(|c| > 5\).

Let's practice!

1. Solve each absolute value inequality and graph the solution set.
   a. \(|a + 5| > 7\)
   b. \(|b - 2| > 1\)
   c. \(|c - 9| > 4\)

What is the solution set for \(c\)?

Let's solve each absolute value inequality and graph the solution set:

For any real numbers \(c\) and \(d\), if \(|c| \leq d\), then \(c \leq d\) or \(c \geq -d\).

For any real numbers \(c\) and \(d\), if \(|c| < d\), then \(-d < c < d\).

Using our definition of absolute value, this is saying that \(c\) represents all the numbers that are \(5\) units from zero on the number line.

Graph all the numbers represented by \(c\) on a number line.
Section 2: Equations and Inequalities

Try It!

3. Solve each equation or inequality and graph the solution set.
   a. \( |2m + 4| \geq 12 \)
   b. \( 2|x| - 4 > 14 \)
   c. \( |p + 7| = -13 \)

2. Tammy purchased a pH meter to measure the acidity of her freshwater aquarium. The ideal pH level for a freshwater aquarium is between 6.5 and 7.5 inclusive.

   a. Graph an inequality that represents the possible pH levels needed for Tammy’s aquarium.
   b. Define the variable and write an absolute value inequality that represents the possible pH levels needed for Tammy’s aquarium.
Section 2: Equations and Inequalities

4. Baseball fans often leave a baseball game if their team is ahead or behind by five runs or more. Toronto Blue Jays fans follow this pattern, and the Blue Jays have scored eight runs in a particular game.

a. Graph an inequality that represents the possible runs, \( r \), scored by the opposing team if Toronto fans are leaving the game.

b. Write an absolute value inequality that represents the possible runs scored by the opposing team if Toronto fans are leaving the game.

BEAT THE TEST!

1. Match the following absolute value equations and inequalities to the graph that represents their solution.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>x + 3</td>
</tr>
<tr>
<td>(</td>
<td>x + 3</td>
</tr>
<tr>
<td>(</td>
<td>x + 3</td>
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<tr>
<td>(</td>
<td>x</td>
</tr>
<tr>
<td>(</td>
<td>x</td>
</tr>
<tr>
<td>(</td>
<td>x</td>
</tr>
</tbody>
</table>

1. Write an absolute value inequality that represents the possible runs if Toronto fans are leaving the game.

2. Write an absolute value inequality that represents the possible runs scored by the opposing team if Toronto fans are leaving the game.

3. Graph an inequality that represents the possible runs scored by the opposing team in a particular game.

4. Baseball fans often leave a baseball game if their team is leading by five runs or more. Toronto Blue Jays fans follow this pattern, and the Blue Jays have scored eight runs in a particular game.
Let’s Practice!

1. Consider the equation $x - 6y = 5p$. Solve for $x$.

2. Consider the equation $2x + 6y = 5p$. Solve for $c$.

Try It!

2. Consider the equation $8 + 6 = 5z$. Solve for $z$.

Sometimes, we might need the formula solved for length.

Consider the formula for the perimeter of a rectangle:

\[ p = 2l + 2w. \]

Did we use different properties when we solved the two equations?

\[ 2z + 4 = 12. \]

Solve each equation for $x$.

Rearranging Formulas

Section 2 – Topic 9

Tip: It is helpful to circle the variable that you are solving for.
3. Consider the equation \(\frac{z}{x - c} = d\). Solve for \(c\).
Section 2: Equations and Inequalities

Section 2 – Topic 10

Solution Sets to Equations with Two Variables

Consider \( x^2 = 5 \). What is the only possible value of \( x \) that makes the equation true?

Now consider \( x + y = 5 \). What are some solutions for \( x \) and \( y \)?

Graph each of the ordered pairs from the previous problem on the graph below.

Possible solutions can be listed as ordered pairs.

Shoe size and foot length for women are related by the formula \( z = 3f - 24 \), where \( f \) represents the length of the foot in inches. Solve the formula for \( f \).

2. What does this formula represent?
Section 2: Equations and Inequalities

What do you notice about the points you graphed?

How many solutions are there to the equation \( 5x + y = 5 \)?

Let's Practice!

1. Tammy has 10 songs on her phone's playlist. The playlist features songs from her two favorite artists, Beyoncé and Pharrell.

a. Create an equation using two variables to represent this situation.

b. List at least three solutions to the equation that you created.

c. Do we have infinitely many solutions to this equation? Why or why not?

d. Create a graph that represents the solution set to your equation.

e. Why are there only positive values on this graph? In this case, our solutions must be natural numbers. Notice that the solutions follow a linear pattern. However, they do not form a line. This is called a discrete function.
Section 2: Equations and Inequalities

3. What if we changed the problem to say the sum of two integers is 15?

a. Create an equation using two variables to represent this situation.

b. Is this function discrete or continuous? Explain your answer.

c. Represent the solution on the graph below.

In this case, our solutions are rational numbers. Notice that the solutions form a line. This is called a continuous function.

Try It!

2. The sum of two numbers is 15.

a. Create an equation using two variables to represent this situation.

b. List at least three possible solutions.

c. Create a visual representation of all the possible solutions on this graph.

d. Is this function discrete or continuous? Explain your answer.

This is called a continuous function. Notice that the solutions form a line. In this case, our solutions are rational numbers.
1. Elizabeth’s tablet has a combined total of 20 apps and movies. Let $x$ represent the number of apps and $y$ represent the number of movies. Which of the following could represent the number of apps and movies on Elizabeth’s tablet? Select all that apply.

- $x = 20$
- $x = 20 + x$
- $y = x$
- $y = y + x$
- $x - y = 0$
- $y - x = 0$
- $x + y = 20$
- $x + y = 20 + y$
- $x + y = 20 + x$
**Section 3: Introduction to Functions**

The following Mathematics Florida Standards will be covered in this section:

- **MAFS.912.A-APR.1.1**
  - Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

- **MAFS.912.A-SSE.1.1**
  - Interpret expressions that represent a quantity in terms of its context.
  - Interpret parts of an expression, such as terms, factors, and coefficients.

- **MAFS.912.A-SSE.1.2**
  - Use the structure of an expression to identify ways to rewrite it.

- **MAFS.912.A-CED.1.3**
  - Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

- **MAFS.912.A-REI.2.3**
  - Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

- **MAFS.912.F-IF.1.1**
  - Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$.

- **MAFS.912.F-IF.1.2**
  - Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

- **MAFS.912.F-IF.2.4**
  - For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

- **MAFS.912.F-IF.2.5**
  - Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

- **MAFS.912.F-IF.3.7.b**
  - Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases.

- **MAFS.912.F-BF.1.1.a,b,c**
  - Write a function that describes a relationship between two quantities.
  - Combine standard function symbols and use the operations of addition, subtraction, multiplication, and division on functions.
  - Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs.

- **MAFS.912.F-IF.5.1.1**
  - Understand the concept of a function and use function notation.

- **MAFS.912.F-LE.1.1**
  - Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$.

- **MAFS.912.F-LE.1.2**
  - Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

- **MAFS.912.F-LE.1.3**
  - Interpret expressions that represent a quantity in terms of its context.
  - Interpret parts of an expression, such as terms, factors, and coefficients.

- **MAFS.912.F-LE.1.4**
  - Interpret expressions that represent a quantity in terms of its context.
  - Interpret parts of an expression, such as terms, factors, and coefficients.

- **MAFS.912.F-LE.1.5**
  - Interpret expressions that represent a quantity in terms of its context.
  - Interpret parts of an expression, such as terms, factors, and coefficients.

- **MAFS.912.F-LE.1.6**
  - Interpret expressions that represent a quantity in terms of its context.
  - Interpret parts of an expression, such as terms, factors, and coefficients.
Consider the following function:

\[ y = 3x^2 + 2x + 1 \]

- **Domain**: The set of all inputs \( x \) for which the function is defined.
- **Range**: The set of all outputs \( y \) calculated from the function.
- **Input and Output Values**: A function is a relationship between input and output.

For every \( x \), there is a unique \( y \).

\( y \) can also be written as \( f(x) \).

In a function, every \( x \) corresponds to only one \( y \).

For the function \( f(x) = 3x^2 + 2x + 1 \),
- **Domain**: The set of all values of \( x \) used for the function.
- **Range**: The set of all values of \( y \) calculated from the function.

Input and Output Values

Section 3 – Topic 1

**MAFS.912.F-BF.2.3**

- **Topics in this Section**
  - Topic 1: Transformations of Functions
  - Topic 2: Understanding Piecewise-Defined Functions
  - Topic 3: Key Features of Graphs of Functions – Part I
  - Topic 4: Multiplying Functions
  - Topic 5: Dividing Functions
  - Topic 6: Closure Property
  - Topic 7: Adding and Subtracting Functions
  - Topic 8: Representing, Naming, and Evaluating Functions
  - Topic 9: Key Features of Graphs of Functions – Part II
  - Topic 10: Understanding Piecewise-Defined Functions
  - Topic 11: Transformations of Functions

- **Even and Odd Functions**
  - Even: \( f(x) = f(-x) \)
  - Odd: \( f(-x) = -f(x) \)

- **Identifying the Effects on the Graph**
- **Finding the Value of \( k \)**

- **Identifying the Effect on the Graph of**
- **Function Transformations**
- **Even and Odd Functions**

- **Key Features of Graphs**

- **Piecewise-Defined Functions**

- **Understanding Piecewise-Defined Functions**

- **Domain**
- **Range**

- **Input and Output Values**
We also refer to the variables as independent and dependent. The dependent variable _______________ the independent variable.

Refer to the mapping diagram on the previous page. Which variable is independent? Which variable is dependent?

Consider a square whose perimeter depends on the length of its sides. What is the independent variable? What is the dependent variable?

We can choose any letter to represent a function, such as $f(x)$ or $g(x)$. By using function notation, such as $f(x)$ or $g(x)$, we show that we are talking about different functions.

Let's Practice!

1. You earn $10.00 per hour babysitting. Your total earnings depend on the amount of hours you spend babysitting.
   a. What is the independent variable?
   b. What is the dependent variable?
   c. How would you represent this situation using function notation?

2. The table below represents a relation.
   a. Is the relation also a function? Justify your answer.
   b. If the relation is not a function, what number could be changed to make it a function?

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

Tip: Study Edge
Section 3: Introduction to Functions

Try It!


a. If Mrs. Krabappel buys 24 composition books, it will cost $30. Write this function using function notation.

b. What is the input and output?

c. Write a function to describe the situation.

d. What does her total cost depend upon?

c. Complete the mapping diagram for c.

b. Complete the mapping diagram for b so that it is NOT a function.

a. Complete the mapping diagram so that it is NOT a function.

Consider the following incomplete mapping diagrams:

Diagram A

Diagram B

Diagram C

Resolving:

The diagram to show a function. If not, justify your answer.

C. Is it possible to complete the mapping diagram for c.?
1. Isaac Messi is disorganized. To encourage Isaac to be more organized, his father promised to give him three dollars for every day that his room is clean and his schoolwork is organized.

Part A: Define the input and output in the given situation.

Input: schoolwork is organized, dollars for every day that his room is clean and his schoolwork is organized.

Output: $3, $6, $9, $12

Part B: Write a function to model this situation.

\[
\begin{align*}
\text{Input} & : x \\
\text{Output} & : f(x)
\end{align*}
\]

2. The cost to manufacture x pairs of shoes can be represented by the function \( C(x) = 63x \). Complete the statements about the function.

\[ C(6) = 378 \]
Section 3: Introduction to Functions

Consider the function $f(x) = \frac{3}{x}$.

What are the values of the domain of $f$?

What are the values of the range of $f$?

Evaluate $f(5) = \frac{3}{5}$ to find the range for the domain $\{ -7, 0, 2 \}$.

Let's Practice!

1. You make trips to the grocery store to purchase doughnuts. Every time you go, you take a taxi. The round-trip taxi ride costs $7.00 and each doughnut costs $0.50.
   a. Write a function to describe the total cost of any given trip.
   b. Write a function to describe the cost of any given tip.
   c. You take the taxi to the store and buy four dozen doughnuts. Represent this situation using function notation and find the total cost.
   d. Your dad gave you $20.00 to buy doughnuts for your friends. You bought him back $2.00. How many dozens of doughnuts did you buy?

Determine whether the following values are true for the given function $f(x)$.

$\frac{32}{3}$ = 32

$\frac{5}{(5)} = f(5)$
Section 3: Introduction to Functions

2. Consider the function \( f(x) = 2x + 1 \). Evaluate \( f(x) \) to find the range for the domain \{-2, -1, 0\}.

Try It!

3. Your sister is using your credit card to buy tickets on TicketBoss for a Taylor Quick concert. There is a 

   a. Write a function to describe the situation.

   b. Evaluate the total cost function if your sister decides to buy seven tickets in a single transaction.

   c. Your credit card statement shows a $225.00 transaction fee with each order, and tickets cost $55.00 each. How many tickets did your sister buy?
### Adding and Subtracting Functions

#### Section 3 – Topic 3

**BEAT THE TEST!**

1. Match the functions in the left column with the values in the right column.

<table>
<thead>
<tr>
<th>Function</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = x^2$</td>
<td>1</td>
</tr>
<tr>
<td>$g(x) = 2x$</td>
<td>2</td>
</tr>
<tr>
<td>$h(x) = x + 1$</td>
<td>3</td>
</tr>
<tr>
<td>$i(x) = 3x$</td>
<td>4</td>
</tr>
<tr>
<td>$j(x) = x^2 - 2x + 1$</td>
<td>5</td>
</tr>
<tr>
<td>$k(x) = 4x + 5$</td>
<td>6</td>
</tr>
<tr>
<td>$l(x) = x^3$</td>
<td>7</td>
</tr>
<tr>
<td>$m(x) = 2x^2 - 3x + 1$</td>
<td>8</td>
</tr>
</tbody>
</table>

#### Adding Functions

Let $f(x) = 2x^2 - 3x + 1$ and $g(x) = x^2 - 2x + 4$.

- $f(x) + g(x) = (2x^2 - 3x + 1) + (x^2 - 2x + 4) = 3x^2 - 5x + 5$

#### Subtracting Functions

Let $f(x) = 2x^2 - 3x + 1$ and $g(x) = x^2 - 2x + 4$.

- $f(x) - g(x) = (2x^2 - 3x + 1) - (x^2 - 2x + 4) = x^2 - x - 3$
Section 3: Introduction to Functions

Let’s Practice

1. Consider the following functions.
   \[ f(x) = 4 \]
   \[ g(x) = 4 \]
   \[ h(x) = (4 - 2x + \sqrt{x}) \]
   \[ x + \sqrt{x} = (x) \]

   a. Find \( h(x) - 2(x) \).
   b. Find \( f(4) \).

2. Recall the functions we used earlier:
   \[ f(x) = 4 \]
   \[ g(x) = 4 \]
   \[ h(x) = (4 - 2x + \sqrt{x}) \]
   \[ x + \sqrt{x} = (x) \]

   a. Let \( m(x) \) be \( f(x) + g(x) \). Find \( m(x) \).
   b. Find \( g'(x) \).

Try It!

1. Consider the following functions.
   \[ f(x) = 4 \]
   \[ g(x) = 4 \]
   \[ h(x) = (4 - 2x + \sqrt{x}) \]
   \[ x + \sqrt{x} = (x) \]

   a. Find \( h(x) - 2(x) \).
   b. Find \( f(4) \).
Consider the functions below.

\[ f(x) = 4x + 2x - 3 \]

Which of the following is the resulting polynomial when \( f(x) \) is subtracted from \( g(x) = 5x + 3x - 4 \)?

A) \( g(x) - f(x) \)
B) \( f(x) - g(x) \)
C) \( 4x - 3 \)
D) \( 4x + 2x - 3 \)

Find the product of the following two functions.

\[ f(x) = 3x + 2 \]
\[ g(x) = 5x - 1 \]

Let \( h(x) = f(x) \cdot g(x) \) and \( x = 2 \). Find \( h(2) \).
Section 3: Introduction to Functions

Let's Practice!

1. Let \( h(x) = \frac{1}{2}x + 5 \) and \( g(x) = 2 \).
   
   Find \( h(g(x)) \).

2. Let \( m(x) = 3x - 2 \) and \( \lambda x = 2x + 3 \).
   
   Find \( m(\lambda x) \).
Section 3: Introduction to Functions

b. What does the function \( f(x) \) represent in this problem?

a. Let \( g(x) = (x)^2 \). Find \( g(x) \).

2. The envelope below has a mailing label:

- POLYNOMIAL, 6X11, INFINITY WAY
- MR. AL GBERA

\[ x^4 + x = (x)^2 \]
\[ 5 + x = 6(x) \]
\[ 5 + 6x = (x)^2 \]
Section 3: Introduction to Functions

1. A square has sides of length \( x \). A rectangle is six inches shorter and eight inches wider than the square.

   **Part A:** Write a function to represent the area of the rectangle in terms of the sides of the square.

   **Part B:** Express both the length and the width of the rectangle as a function of the sides of the square.

2. Felicia needs to find the area of a rectangular field in her backyard. The length is represented by the function \( L(x) = x + 1 \) and the width is represented by the function \( W(x) = 3x^2 + 6 \). The resulting expression for \( A(x) = L(x) \cdot W(x) \) is a...
Let's discuss an important fact that we need to be aware of when dividing.

Consider \( \frac{32}{4} \).

What is another way of expressing this equation?

Consider \( \frac{2}{4} \).

What is another way of expressing this equation?

Consider \( \frac{4}{2} \).

Can you write an equivalent expression?

The quotient of any real number and zero is always undefined.

Consider \( \frac{0}{2} \).

Therefore, when we are dividing functions or polynomials and we have expressions with variables in the denominator, we need to:

1. Determine what value(s) of the variable would make the denominator equal zero.
2. Rewrite the equation or expression stating the constraints.

Therefore, when we are dividing functions or polynomials and when dividing.

Let's discuss an important fact that we need to be aware of when dividing.

Dividing Functions
Section 3: Introduction to Functions

Let $\# = \#G$, $\# = \#$, and $(\# = \#4G$.

Rewrite the following as an equation and state the constraints.

For what value of $\#$ does $\# = \#$?

Let's Practice!

1. Let $\# = \#$, $\# = \#F$, and $\# = \#G$.

   a. Find the value of $\#$ for which $\# = \#(\#)$.

   b. Find the value of $\#$ for which $\# = \#(\#)$ and $\# = \#(\#) = \#(\#) = \#(\#)$.

   c. Find the value of $\#$ for which $\# = \#(\#)$.

Rewrite the following as an equation and state the constraints.

Let $\# = \#$, $\# = \#9$, and $\# = \#16$. Let $\#$ and $\# = \#$.
Try It!

2. Let $x = (x)^a$ and $y = (x)^b$, and $z = (x)^c$.

a. What is the difference between the domain of $f(x)$ and the domain of $g(x)$?

b. Find the value of $x$ for which $(x)^d = (x)^e$.

c. Find the value of $x$ for which $(x)^f = (x)^g$.

d. Find the value of $x$ for which $(x)^h = (x)^i$.

Part A: For what value of $x$ does $f(x) = 3$?

Part B: For what value of $x$ does $f(x) = 5$ and state the constraint(s).
Section 3: Introduction to Functions

2. Consider the following function.

\[-G(-4) \neq (-4G)(-F)\]

Which of the following values of \(-G\) are excluded? Select all that apply.

- G
- 4
- G
- 2
- 0
- -2
- -3
- -9

Section 3 – Topic 6

Closure Property

When we add two integers, what type of number is the sum?

When we multiply two irrational numbers, what type of number is the product?

Are integers closed under addition? Justify your answer.

Are irrational numbers closed under multiplication? Justify your answer.

Would integers be closed under division? Justify your answer.

Which of the following values of \(r\) are excluded? Select all that apply.

\[\frac{2(1)(r)(6) - 9(1)}{8(r)(16 - 9)} = (2)\]

Consider the following function.
Section 3: Introduction to Functions

Let's Practice!

1. Check the boxes for the following sets that are closed under the given operations.

<table>
<thead>
<tr>
<th>Set</th>
<th>+</th>
<th>−</th>
<th>×</th>
<th>÷</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rational numbers</td>
<td>☑</td>
<td>☑</td>
<td>☑</td>
<td>☑</td>
</tr>
<tr>
<td>Polynomials</td>
<td>☑</td>
<td>☑</td>
<td>☑</td>
<td>☑</td>
</tr>
<tr>
<td>Integers</td>
<td>☑</td>
<td>☑</td>
<td>☑</td>
<td>☑</td>
</tr>
</tbody>
</table>

Example: Let's apply the closure property to polynomials.

Polynomials are closed under addition.

Are the following statements true or false? If false, give a counterexample.

- Polynomials are closed under subtraction.
- Polynomials are closed under multiplication.
- Polynomials are closed under division.
Section 3: Introduction to Functions

Try It!

1. Ms. Sanabria claims that the closure properties for polynomials are analogous to integers. Mr. Roberts claims that the closure properties for polynomials are analogous to rational numbers. Who is correct? Explain your answer.

BEAT THE TEST!

2. Choose from the following words and expressions to complete the statement below.

The product of $5x^4 - 3x^2 + 2$ and ______________________ illustrates the closure property because the ______________________ of the product are ______________________ and the product is a polynomial.

Complete the statement below.

1. **Exponents**
2. **Coefficients**
3. **Rational numbers**
4. **Variables**
5. **Whole numbers**
6. **Integers**

"New" Sandhing claims that the closure properties for polynomials are analogous to integers. Mr. Roberts claims ____________________
Section 3: Introduction to Functions

2. The freshman class is selling t-shirts to raise money for a field trip. The cost of the t-shirt and design is $8, with a $4.50 setup fee. The class plans to sell the shirts for $12.

a. Define the variable.
b. Write a cost function.
c. Write a revenue function.
d. Write a profit function.

Try It!

Let's Practice!

1. At the fall festival, the senior class sponsors hayrides to raise money for the senior trip. The ticket price is $5, and each hayride carries an average of 15 people. They consider raising the ticket price in order to earn more money. For each $0.50 increase in price, an average of 2 fewer seats will be sold. Let x represent the number of $0.50 increases.

a. Write a function, c(x), to represent the cost of one ticket based on the number of increases.
b. Write a function, r(x), to represent the number of riders based on the number of increases.
c. Write a revenue function for the hayride that could be used to maximize revenue.

There are many times in real-world situations when we must combine functions. Profit and revenue functions are a great example of this.

Functions

Real-World Combinations and Compositions of Functions
Section 3: Introduction to Functions

Let's Practice!

3. Priscilla works at a cosmetics store. She receives a weekly salary of $500 and is paid a 3% commission on weekly sales over $3500. A landscaping company installed a circular sprinkler system. The water reaches its maximum radius of 10 feet after 30 seconds. The company wants to know the area covered by the sprinkler after 30 seconds. The company wants to know the area covered by the sprinkler after 30 seconds.

a. Let \( t \) represent the time in seconds after the sprinkler is turned on. Write a function, \( A(t) \), to represent the area covered by the sprinkler after \( t \) seconds.

b. Let \( r \) represent the radius of the sprinkler system. Write a function, \( r(t) \), to represent the radius of the sprinkler system after \( t \) seconds.

c. Write a composite function, \( A(r(t)) \), to represent the area covered by the sprinkler system after \( t \) seconds.
Part A: Write the function $g(x)$.

Part B: Match each of the following to what they represent. Some letters will be used twice.

A. The cost of the furniture, sales tax, and delivery fee.
B. The cost of the furniture and delivery fee.
C. The cost of the furniture and sales tax.

Part C: Write the function $f(x)$.

The following functions represent the situation:

1. A furniture store charges 6.5% sales tax on the cost of the furniture and a $20 delivery fee. The delivery fee is not subject to sales tax. (The delivery fee is not included in the cost of the furniture and sales tax.)

Let's review the definition of a function.

Consider the following graph.

Determine if the graph above represents a function. We call this the vertical line test. Use the vertical line test to determine if a graph represents a function.

How can a vertical line help us quickly determine if a graph represents a function?
Section 3: Introduction to Functions

1. Use the vertical line test to determine if the following graphs are functions.

2. Which of the following represent a function? Select all that apply.

Try It!

Sketch a graph of a function that is not linear.

Functions are not always linear.

Describe a graph of a line that is not a function.

Graphs of lines are not always functions. Can you

Important facts:

Let's Practice!
Section 3: Introduction to Functions

It's important to understand key features of graphs.

The $y$-intercept of a graph is the location where the graph crosses the $y$-axis.

The $x$-coordinate of the $y$-intercept is always $0$.

The $x$-intercept of a graph is the location where the graph crosses the $x$-axis.

The $y$-coordinate of the $x$-intercept is always $0$.

The $y$-intercept is the point on a function or graph.

It's important to understand key features of graphs.

World context:

Each of these features are very helpful in understanding real-world context.

Consider the following scenarios. Determine if each one represents a function or not.

a. An analyst takes a survey of people about their heights, in inches, and their ages and then relates their heights to their ages.

b. A geometry student is dilating a circle and analyzes the area of the circle as it relates to the radius.

c. A teacher has a roster of 32 students and relates the students' letter grades to the percent earned.

d. A boy throws a tennis ball in the air and lets it fall to the ground. The boy relates the time passed to the height of the ball.
5. Refer to the previous problem for the following questions.

b. What is the \( y \)-intercept represent in this real world context?

c. What is the solution to this situation?

d. What is the \( x \)-intercept represent in this real world context?

c. Label the intercepts on the graph.

4. Consider the following graph that represents the height in feet, of a water balloon dropped from a 2nd story window after a given number of seconds.

Try it!
Key Features of Graphs of Functions – Part 2

Let's discuss other key features of graphs of functions.

1. Use the following graph of an absolute value function to answer the questions below.

<table>
<thead>
<tr>
<th>a. Define the domain.</th>
</tr>
</thead>
<tbody>
<tr>
<td>b. Define the range.</td>
</tr>
<tr>
<td>c. Where is the graph increasing?</td>
</tr>
<tr>
<td>d. Where is the graph decreasing?</td>
</tr>
<tr>
<td>e. Identify any relative maxima.</td>
</tr>
<tr>
<td>f. Identify any relative minima.</td>
</tr>
</tbody>
</table>

Tip: We read a graph from left to right, like reading a book.

Domain: the input or the values.

Range: the __________________ or the values.

Increasing intervals: as the #values __________________, the %values _______________.

Decreasing intervals: as the #values __________________, the %values _______________.

Relative maximum: the point on a graph where the interval changes from __________________ to __________________.

Relative minimum: the point on a graph where the interval changes from __________________ to __________________.

Let's Practice!
3. Describe everything you know about the following graph of an exponential function.

2. Use the graph of the following quadratic function to answer the questions below.

a. Define the domain.

b. Define the range.

c. Where is the graph increasing?

d. Where is the graph decreasing?

e. Identify any relative minimums.

f. Identify any relative maximums.
Understanding Piecewise-Defined Functions

What is a piecewise function?

A function made up of distinct "pieces" based on different rules for the values.

The pieces of a piecewise function are graphed together on the same coordinate plane.

Describe an example of a piecewise function used in our daily lives.

Since it is a function, it will pass the vertical line test.

The domain is the x-values, or input.

The range is the y-values, or output.

The graph is increasing when the domain is

The graph is decreasing when the graph is increasing when

The graph has one relative minimum when

There is a relative minimum at (2, 2).

The range is {y | 0 < y ≤ 5,}

The range is {y | y < 2,}

The graph is decreasing when the domain is

The graph is increasing when

The graph is increasing when

The range is $R = \{y | 0 < y \leq 5, \}$

Which of the following statements are true about the graph? Select all that apply.

The graph is increasing when $x < 4$.

The graph is increasing when $4 \leq x < 5$.

The graph has one relative minimum.

The graph is increasing when $x > 4$.

The graph is decreasing when the domain is $x < 1$.

There is a relative minimum at $(2, 2)$.

The following graph is a piecewise function.
Let's note some of the features of the graph:

- The domain of the piecewise graph can be represented with intervals. If we define the first interval as \( M \), the second interval would be ____________________.

- The graph is nonlinear (curved) when the domain is ____________________.

- The graph is linear when the domain is ____________________.

- There is one closed endpoint on the graph, which means that the particular domain value, zero, is ____________________ in that piece of the function. This illustrates the inclusion of zero in the function.

- There is one open circle on the graph, which means that the particular domain value, zero, is ________   _____________________ in that piece of the function. This illustrates the constraint that \( L \) for the function __________________.

---

Consider the following piecewise-defined function:

\[
0 < x \leq 2, \text{ when } x^2 + 1, \quad \text{and} \quad 0 \leq x < 2, \text{ when } x^2 - x^2 = (x) f
\]
Section 3: Introduction to Functions

f(x) is a special type of piecewise function known as a ________________ function, which resembles a series of steps.

Step functions pair every ___________ value in a given ___________ (particular section of the ____________) with a single value in the range (________-value).

Try It!

2. Consider the previous graph in exercise 1.
   a. How many pieces are in the step function? Are the pieces linear or nonlinear?
   b. How many intervals make up the step function? What are the interval values?
   c. Why are open circles used in some situations and closed circles in others?
   d. How do you know this is a function?
   e. What is the range of this piecewise function?
Section 3: Introduction to Functions

Part A: Piecewise-defined functions are represented by
- at least two functions
- at least one function
- domain values
- range values

Correct answers from each box:
- \( x \)
- \( f(x) \)

Part B: When evaluating piecewise-defined functions,
- choose which equation to use based on the
- choose which equation to use based on the
- slope
- x-value
- constant

Using your instincts:
- order of operations
- exponent rules

and then substitute and evaluate.

1. Evaluate the piecewise-defined function for the given values of \( x \) by matching the domain values with the range values.

\[
\begin{align*}
& x < 4 \\
& 4 \leq x \leq 2 \\
& 2 < x \\
\end{align*}
\]

\[
\begin{align*}
& x = 2 \\
& x = 3 \\
& x = 4 \\
\end{align*}
\]

\[
\begin{align*}
& f(x) = \begin{cases} 
3 & \text{if } x < 4 \\
2x - 1 & \text{if } 4 \leq x \leq 2 \\
-x + 8 & \text{if } 2 < x \\
\end{cases}
\end{align*}
\]
Which graphs transform the dependent variable?

Which graphs transform the independent variable?

Describe what happened in each graph.

The graphs on the following page are transformations of $f(x)$.

The graph of $f(x)$ is shown below.
Section 3: Introduction to Functions

Let's Practice!

1. In the following functions, state whether the independent or dependent variable is being transformed and describe the transformation (assume \( a \neq 0 \)).

<table>
<thead>
<tr>
<th>( f(x) )</th>
<th>( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td>0.5</td>
</tr>
<tr>
<td>( x \theta )</td>
<td>( x \theta )</td>
</tr>
</tbody>
</table>

The function \( y(x) = (2x \theta)^\beta \). Complete the table for \( y(x) \).

2. The following table represents the function \( g(x) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>0.5</td>
</tr>
<tr>
<td>-2</td>
<td>0.25</td>
</tr>
<tr>
<td>( x \theta )</td>
<td>( x \theta )</td>
</tr>
</tbody>
</table>

The function \( h(x) = (2x \theta)^\beta \). Complete the table for \( h(x) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>0.5</td>
</tr>
<tr>
<td>-2</td>
<td>0.25</td>
</tr>
<tr>
<td>( x \theta )</td>
<td>( x \theta )</td>
</tr>
</tbody>
</table>

In the following functions, state whether the independent variable is being transformed and describe the transformation (assume \( a \neq 0 \)).

\[
\begin{align*}
(\gamma - x)f(x) &= c \\
(\gamma + x)f(x) &= b \\
\gamma f(x) &= a
\end{align*}
\]
Section 3: Introduction to Functions

3. The table below shows the values for the function \( f(x) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>( z )</td>
</tr>
<tr>
<td>( z )</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( -1 )</td>
<td>1</td>
</tr>
<tr>
<td>( -2 )</td>
<td>( x )</td>
</tr>
</tbody>
</table>

4. The graph of \( f(x) \) is shown below.

Let \( g(x) = f(x) = x^2 + 3 \) - 2.

Graph \( g(x) \) on the coordinate plane with \( f(x) \).
Section 3: Introduction to Functions

2. The table below shows the values for the function $f$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
</tr>
</tbody>
</table>

Complete the table for the function $f$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$x-3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$x-3$</td>
</tr>
</tbody>
</table>

Graph the function $f(x)$ on the coordinate plane with $f(x)$.

Let $f(x) = (x)(x)$ and $g(x) = (x-x)$. Then

1. The graph of $f(x)$ is shown below.

2. The table below shows the values for the function $p(x)$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$p(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>Standard</td>
<td>Description</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>MAFS.912.F.1.3</td>
<td>Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.</td>
</tr>
<tr>
<td>MAFS.912.F.2.6</td>
<td>Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.</td>
</tr>
<tr>
<td>MAFS.912.F.1.2</td>
<td>Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input/output pairs (including reading these from a table). This section focuses on linear functions.</td>
</tr>
<tr>
<td>MAFS.912.A.CED.1.2</td>
<td>Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</td>
</tr>
<tr>
<td>MAFS.912.A.CED.1.1</td>
<td>Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process or steps for calculation from context.</td>
</tr>
<tr>
<td>MAFS.912.A.REI.3.5</td>
<td>Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.</td>
</tr>
<tr>
<td>MAFS.912.A.REI.3.6</td>
<td>Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</td>
</tr>
<tr>
<td>MAFS.912.A.REI.4.11</td>
<td>Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately (e.g., using technology) if the functions are linear, polynomial, rational, or absolute value.</td>
</tr>
<tr>
<td>MAFS.912.A.REI.4.10</td>
<td>Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).</td>
</tr>
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<td>MAFS.912.A.REI.4.10</td>
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</tr>
<tr>
<td>MAFS.912.A.REI.4.10</td>
<td>Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).</td>
</tr>
</tbody>
</table>
Section 4: Linear Functions

MAFS.912.A.REI.4.12: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

MAFS.912.S-ID.3.7: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Topics in this Section

**Topic 1:** Arithmetic Sequences

Let's look at the following sequence of numbers:

\[ 3, 8, 13, 18, 23, \ldots \]

This is a sequence where each term is the \[ \_ \] of the previous term and a common difference, \[ \_ \].

This is an example of an arithmetic sequence.

There are 5 terms in this sequence so far:

- 3 is the 1st term
- 8 is the 2nd term
- 13 is the 3rd term
- 18 is the 4th term
- 23 is the 5th term

The „…” at the end means that this goes on forever.

Let's look at the following sequence of numbers:

**Section 4 – Topic 1: Arithmetic Sequences**

MAFS.912.S-ID.3.7: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

MAFS.912.A.REI.4.12: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
We can represent this sequence in a table:

<table>
<thead>
<tr>
<th>Term</th>
<th>Function Notation</th>
<th>Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1)</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>(2)</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>(3)</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>(4)</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>(5)</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>(6)</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>(7)</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>(8)</td>
<td>8</td>
</tr>
</tbody>
</table>

This is a recursive formula.

We can represent this sequence in a table:

<table>
<thead>
<tr>
<th>Term</th>
<th>Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>6</td>
<td>17</td>
</tr>
<tr>
<td>7</td>
<td>21</td>
</tr>
<tr>
<td>8</td>
<td>25</td>
</tr>
</tbody>
</table>

How can we find the 9th term of this sequence?

One way is to start by finding the previous term:

<table>
<thead>
<tr>
<th>Term</th>
<th>Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>29</td>
</tr>
</tbody>
</table>

Can you think of a situation where the recursive formula would take a long time to use?

In order to solve for a term, you must know the value of its preceding term.

This is a recursive formula.
Let's look at another way to find unknown terms:  

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Sequence Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>$F(6)$</td>
</tr>
<tr>
<td>7</td>
<td>$F(7)$</td>
</tr>
<tr>
<td>8</td>
<td>$F(8)$</td>
</tr>
<tr>
<td>9</td>
<td>$F(9)$</td>
</tr>
</tbody>
</table>

Write a general equation that we could use to find any term in the sequence. This is an explicit formula.  

To solve for a term, you need to know the first term of the sequence and the difference by which the sequence is increasing or decreasing.  

Let's Practice!  

1. Consider the sequence $G, G, \ldots$  
   a. Write a recursive formula for the sequence.  
   b. Write an explicit formula for the sequence.  
   c. Find the $3^{rd}$ term of the sequence.

Try It!  

2. Consider the sequence $\ldots$  
   a. Find the next three terms of the sequence.  
   b. Write a recursive formula for the sequence.  
   c. Write an explicit formula for the sequence.  
   d. Find the $3^{rd}$ term of the sequence.
Section 4: Linear Functions

Part A:
If she starts with five walking planks on the first day,完成on any given day.
find the number of walking planks Yohanna completes on the second, third, and fourth days. She will complete four more walking planks than the previous day. This is an example of a linear function.

Part B:
How many walking planks will Yohanna do on the 12th day?

Graph 1: Genesis' Reading
Graph 2: July's Reading

- Graph 1: Genesis reads 16 pages of the fault in our stars every day.
- Graph 2: July reads 8 pages every day of the same book.

Represent both situations on the graphs below using the same scales on the axes for both graphs.

Yohanna is conditioning all summer to prepare for her high school's varsity soccer team tryouts. She is incorporating walking planks into her daily workout training plan. Every walking plank is worth one point in our stars every day.

Yohanna completes four more walking planks than the previous day. She will complete four more walking planks than the previous day. This is an example of a linear function.

1. Yohanna is conditioning all summer to prepare for her high school's varsity soccer team tryouts. She is incorporating walking planks into her daily workout training plan. Every walking plank is worth one point in our stars every day.

Part A: Write an explicit formula that can be used to find the number of walking planks Yohanna completes on any given day.

Part B: How many walking planks will Yohanna do on the 12th day?
What is the slope of the following graphs? What does the slope represent?

We can also find slope by looking at the change in \( y \) over the change in \( x \). This is also called the slope of the line.

Graph 4: Jacob's
Graph 3: Aaron's
Graph 2: per
Graph 1: per

What is the rate of change for each of the graphs?

In each of the graphs, we were finding the rate of change in the given situation.

Represent both situations on the graphs below using the same scales for both graphs.

Jacob loves Cherry Coke. Each mini-can contains 100 calories.

Aaron loves Cherry Coke. Each mini-can contains 1000 calories.
Section 4: Linear Functions

b. What does the slope of the line represent?

a. What is the slope of the line of best fit?

Let's Practice!

1. Consider the following graph.

2. Freedom High School collected data on the GPA of various students and the number of hours they spent studying each week. A scatterplot of the data is shown below with the line of best fit.
Section 4: Linear Functions

Part A: Represent the situation on the graph below.

Part B: What does the role of change represent?

A cleaning service cleans many apartments each day. The following table shows the number of hours the cleaners spend cleaning and the number of apartments they clean during that time. The following table shows the number of hours the cleaning company cleans each day.

<table>
<thead>
<tr>
<th>Apartments Cleaned</th>
<th>Time (Hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

a. What is the role of change?

b. What does the role of change represent?

3. Sarah’s parents give her $100.00 allowance at the beginning of each month. Sarah spends her allowance on comic books. The graph below represents the amount of money Sarah spent on comic books last month.
Part B: The data suggest a linear relationship between the number of hours spent cleaning and the number of apartments cleaned. Assuming the relationship is linear, what does the rate of change represent in the context of this relationship?

A. The number of apartments cleaned after one hour.
B. The number of hours it took to clean one apartment.
C. The number of apartments cleaned each hour.
D. The number of apartments cleaned before the company started cleaning.

Part C: Which equation describes the relationship between the time elapsed and the number of apartments cleaned?

A. $2x + y = 2$
B. $x = y$
C. $2x + y = 2$
D. $x = y$

Section 4 – Topic 3

Interpreting Rate of Change and $y$-Intercept

In a Real-World Context – Part 1

Relationship: A linear relationship is linear, what does the rate of change represent in the context of this relationship? The number of hours spent cleaning and the number of apartments cleaned. Assuming the number of apartments cleaned, what does the rate of change represent in the context of this relationship?
Section 4: Linear Functions

What is the slope of the line? What does the slope represent?

At what point does the line intersect the $y$-axis? What does this point represent?

This point is the $y$-intercept of a line.

Let's Practice!

1. You saved $250.00 to spend over the summer. You decide to budget $25.00 to spend each week. 

   a. Define the variable and write a function that represents this situation.
   b. Represent the situation on a graph.
   c. What is the slope of the line? What does the slope represent?
   d. What is different about the slope of this line compared to our earlier problem? Why is it different?
   e. What is the $y$-intercept? What does this point represent?

At what point does the line intersect the $y$-axis? What does this point represent?
Consider the three functions that you wrote regarding the cab ride, summer spending habits, and the community pool membership.

What do you notice about the constant term and the coefficient of the x term?

The constant term is the ______________.

The coefficient of the x is the ______________.

These functions are written in **slope-intercept form**.

We can use slope-intercept form to graph any linear equation.

The coefficient of the x is the slope and the constant term is the “y-intercept ONLY if the equation is in slope-intercept form.

What is the y-intercept? What does the y-intercept represent?

What is the slope of the line? What does the slope represent?

What does each point represent?

Try It!
Section 4: Linear Functions

1. Graph $y = 2x + 3$.

Let's Practice

In a Real-World Context - Part 2
Interpreting Rate of Change and Y-Intercept

2. Consider the equation $2x + 5y = 1$.

b. Rewrite the equation in slope-intercept form.

c. Identify the slope and y-intercept.

d. Graph the equation.
Section 4: Linear Functions

Which of the following statements are true? Select all that apply.

- Line $t$ represents a discrete function.
- The $y$-intercept of line $t$ is 2.
- The slope of line $t$ is equal to $\frac{CD}{AF}$.
- The slope of line $t$ is equal to $\frac{AC}{EC}$.
- The slope of line $t$ is equal to the slope of line $p$.
- The slope of line $t$ is equal to the slope of line $q$.

1. Line $t$, $ACD$, and $APAB$ are shown on the coordinate grid below.

BET THE TEST!
Section 4: Linear Functions

1. Introduction to Systems of Equations

2. Topic 5

Consider the following systems of equations:

**Line 1:** \(2x - y = -5\)

**Line 2:** \(2x + y = 1\)

Graph the system of equations on the coordinate plane.

Recall that a solution to a linear equation is any ordered pair that makes that equation true.

**Part A:** How much does one ticket cost?

**Part B:** How much money did the senior class have at the start of the fundraiser?

2. The senior class at Elizabeth High School was selling tickets.
Let's Practice!

1. Consider the following system of equations made up of Line 1 and Line 2.

   Line 1: $5x + 2y = 8$
   Line 2: $-3x - 2y = -4$

   a. The ordered pair $(2,5)$ is a solution to
   b. The ordered pair $(2,-1)$ is a solution to
   c. The ordered pair $(0,4)$ is a solution to

What do you notice about the point $(1,1)$? What do you notice about the point $(1,3)$? What do you notice about the point $(1,7)$? What do you notice about the point $(2,5)$?
Section 4: Linear Functions

2. Is there ever a time when a system of equations will not have a solution? If so, sketch an example.

3. Consider the following system of equations.

   \[ \begin{align*}
   2x - 5y &= -5 \\
   4x + 3y &= 3
   \end{align*} \]

   a. Graph the system of equations.

   b. What can be said about the solution to this system of equations?

4. Consider the following system of equations.

   \[ \begin{align*}
   -2x + 2y &= -6 \\
   x &= y - 3
   \end{align*} \]

   a. Sketch the graph of the system of equations.

   b. What can be said about the solution to this system of equations?
Section 4: Linear Functions

Finding Solution Sets to Systems of Equations

Part A: Sketch the graph of the system of equations.

\[ z - 2 = y - x \]
\[ 5 = y + x \]

Part B: Determine the solution to the system of equations.

Part C: Create a third equation that could be added to the system so that the solution does not change.

Using Substitution and Graphing

Let's Practice!

1. Brianna’s lacrosse coach suggested that she practices yoga to improve her flexibility. “Yoga-ta Try This!” Yoga Studio has two membership plans. Plan A costs $100.00 per month plus $10.00 per class. Plan B costs $20.00 per month for unlimited classes. Plan A costs $10.00 per class.

a. Define a variable and write two functions to represent the monthly cost of each plan.

The monthly cost of each plan.

\[ \text{Plan A: } C_A = 100 + 10x \]
\[ \text{Plan B: } C_B = 20 + 0x \]

b. Determine the solution to the system of equations.

There are many times that we are able to use systems of equations to solve real-world problems. One method of solving systems of equations is by graphing like we did in the previous video.
Section 4: Linear Functions

2. Brianna is trying to determine which plan is more appropriate for the number of classes she wants to attend.

a. When will the two plans cost exactly the same?

b. When is plan A the better deal?

c. When is plan B the better deal?

We can also help Brianna determine the best plan for her without graphing. Consider our two equations again. We simply want to know when the total costs would be equal.

This method is called solving by _____________.

Set the two plans equal to each other and solve for the number of visits.

<

<

We can also help Brianna determine the best plan for her without graphing, Consider our two equations again. We simply want to know when the total costs would be equal.

What do the Y-intercepts of the graphs represent?

What does the role of change represent in this situation?

What is the rate of change for each plan?

Represent the two situations on the graph below.
Section 4: Linear Functions

4. What is the rate of change of each line? What do they represent?

c. Describe a situation when renting from Vespa Scooter Rental would be a better deal than renting from Scottie’s Scooter Rental.

b. Describe a situation when renting from Vespa Scooter Rental.

a. When will renting a scooter from Vespa Scooter Rental cost the same amount as renting from Scottie’s Scooter Rental?

It's difficult to find the solution by looking at the graph. In such cases, it's better to use substitution to solve the problem.

4. Use the substitution method to help the renter determine when the two scooter rentals will cost the same amount.

a. When will renting a scooter from Vespa Scooter Rental cost the same as renting a scooter from Scottie’s Scooter Rental?

b. Describe a situation when renting from Vespa Scooter Rental would be a better deal than renting from Scottie’s Scooter Rental.

Vespa Scooter Rental rents scooters for $45.00 and $30.25 per mile. Scottie’s Scooter Rental rents scooters for $35.00 and $30.30 per mile.

Try this.

They represent:

d. What do the -intercepts of each line represent?

c. What is the rate of change of each line? What do they represent?
Section 4: Linear Functions

11

Section 4 – Topic 7

Using Equivalent Systems of Equations

An ordered pair that satisfies all equations in a system is called the solution to that system. If two systems of equations have the same solution, they are called equivalent systems.

Consider the following system of equations:

\[
6 = x - y \\
4 = x + y
\]

The solution to this system is \((5, -1)\). We can also see this when we graph the lines.

Part A: After how many weeks of making the additional deposits will Lyle have the same amount of money as Shaun?

Part B: Which method would you use to solve the problem, substitution or graphing? Explain your answer.

Part C: After how many weeks of making the additional deposits will Lyle have the same amount of money as Shaun and Shaun have in their accounts.

Lyle and Shaun open a savings account at the same time. Lyle deposits \(\$500\) initially and adds \(\$10\) per week. Shaun deposits \(\$100\) initially and adds \(\$20\) per week. Shaun wants to know when Lyle will have the same amount in her account as Lyle.

1. Why and Shaun open a savings account at the same time.
Section 4: Linear Functions

Algebraically, show that (5, -1) is also a solution to the sum of the two lines.

\[
\begin{align*}
g &= \lambda - x \\
\theta &= \lambda + x
\end{align*}
\]

Consider the original system of equations again.

Use the process to write an equivalent system.

Describe the result when we multiply either of the equations by some factor.
Section 4: Linear Functions

Let's revisit the original system:

Equation 1:

Equation 2:

Complete the following steps to show that replacing one equation by the sum of that equation and a multiple of the other equation produces a system with the same solutions.

Create a third equation by multiplying Equation 1 by two.

Create a fourth equation by finding the sum of the third equation and Equation 2.

Equation 2: \( z = x - 6 \)

Equation 1: \( z = x + 4 \)

Let's revisit the original system.

Graph the new equation on the same coordinate plane with our original system.

What is the resulting equation when we subtract the second equation from the first equation?
Section 4: Linear Functions

Let's Practice!

1. Consider the following system, which has a solution of \((5, -1)\) and \((7, 2)\), and are non-zero real numbers:

\[
\begin{align*}
L &= \lambda S + \lambda R \\
F &= \lambda N + \lambda M
\end{align*}
\]

Write two new equations that could be used to create an equivalent system of equations.

2. Try It!

List three ways that we can write new equations that can be used to create an equivalent system of equations.

Algebraically, show that \((5, -1)\) is a solution to the fourth equation.

\[
\begin{align*}
\text{Consider the following system, which has a solution of } (5, -1) \\
\text{Graph the fourth equation on the same coordinate plane with}
\end{align*}
\]
Section 4: Linear Functions

Section 4 – Topic 8

Finding Solution Sets to Systems of Equations

Using Elimination

Describe what the graph of the two systems would look like.

Determine the solution to the system of equations.

Consider the following system of equations:

Write an equivalent system that will eliminate one of the variables when you add the equations.

This method of solving a system is called ____________________.

Describe what the graph of the two systems would look like.

Consider the following system of equations:

Write an equivalent system that will eliminate one of the variables when you add the equations.

The system

has the solution \((1, -3)\), where

The system
Let’s Practice!

1. Ruxin and Andre were invited to a Super Bowl party. They were asked to bring pizzas and sodas. Ruxin brought three pizzas and four bottles of soda and spent $67.25. Andre brought five pizzas and two bottles of soda and spent $48.05. Andre was asked to bring pizzas and sodas. Ruxin brought three pizzas and one soda. Andre brought four pizzas and three bottles of soda.

   a. Write a system of equations to represent the situation.

   b. Write an equivalent system that will eliminate one of the variables when you add the equations.

   c. Solve the system to determine the cost of one pizza and one soda.
Section 4: Linear Functions

Try It!

2. Jazmin and Justine went shopping for back to school clothes. Jazmin purchased three shirts and one pair of shorts and spent $68. Justine bought four shirts and three pairs of shorts and spent $82.

a. Assuming all the shirts cost the same amount and all the shorts cost the same amount, write a system of equations to represent each girl's shopping spree.

b. Use the elimination method to solve for the price of one pair of shorts.

<table>
<thead>
<tr>
<th>System</th>
<th>Equation 1</th>
<th>Equation 2</th>
<th>Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jazmin</td>
<td>$6x + 9y = 92$</td>
<td>$-6x + 6y = 54$</td>
<td>Divide by 17.</td>
</tr>
<tr>
<td></td>
<td>$68 = 8y + 82$</td>
<td>$-6x + 6y = 54$</td>
<td>Solve for $x$.</td>
</tr>
</tbody>
</table>

Complete the following table.

1. Solve by elimination:

   \[ 3x + 4y = 46 \]
   \[ 2x - 3y = 8 \]

2. Jazmin and Justine went shopping for back to school clothes. Jazmin purchased three shirts and one pair of shorts and spent $68. Justine bought four shirts and three pairs of shorts and spent $82.

   a. Assuming all the shirts cost the same amount and all the shorts cost the same amount, write a system of equations to represent each girl's shopping spree.

   b. Use the elimination method to solve for the price of one pair of shorts.

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</tr>
</tbody>
</table>

Complete the following table.

1. Solve by elimination:

   \[ 3x + 4y = 46 \]
   \[ 2x - 3y = 8 \]
Section 4: Linear Functions

Graph the line \( y = 2x - 1 \). Do you notice about the solutions to the inequality \( y \geq 2x - 1 \) in the same coordinate plane? What do you notice about the solutions to the inequality \( y \geq 2x - 1 \)?

Plot each solution as a point \((x', y')\) in the coordinate plane:

\( (0, 5), (1, 3), (1.5, 1), (2.5, 0.5), (4, 4) \)

Underline each ordered pair \((x', y')\) that is a solution to the inequality \( y \geq 2x - 1 \).

Consider the following linear inequalities:

\[ \begin{align*}
-4x + 8y &= -10 \\
12x + 8y &= 16 \\
6x + 12y &= -30 \\
18x + 12y &= 72 \\
6x + 8y &= -20 \\
4x + 4y &= 24 \\
2x - 4y &= -10 \\
6x + 4y &= 24
\end{align*} \]

Which of the systems of equations below could not be used to solve the following system for \( x \) and \( y \)?
Section 4: Linear Functions

1.

Let's Practice!

1. The senior class is raising money for Grad Bash. The students' parents are donating cakes. The students plan to sell entire cakes for $5.00 each and slices of cake for $2.00 each. If they want to raise at least $500.00, how many of each could they sell?

   a. List two possibilities for the number of whole cakes and cake slices students could sell to reach their goal of raising at least $500.00.

   b. Write an inequality to represent the situation.

   c. Graph the region where the solutions to the inequality would lie.
Section 4: Linear Functions

1. Try It!

2. The freshman class wants to include at least 120 people in the pep rally. Each skit will feature 20 people, and the dance routines will feature 15 people.

a. List two possible combinations of skits and dance routines.

b. Write an inequality to represent the situation.

c. Graph the region where the solutions to the inequality lie.

d. What does the y-intercept represent?
### Part B: Determine whether these combinations of basketballs, \( b \), and soccer balls, \( s \), can be purchased.

<table>
<thead>
<tr>
<th>( b )</th>
<th>( s )</th>
<th>( b = 3 )</th>
<th>( b = 2 )</th>
<th>( b = 1 )</th>
<th>( b = 0 )</th>
<th>( b = -1 )</th>
<th>( b = -2 )</th>
<th>( b = -3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s = 0 )</td>
<td>( s = 1 )</td>
<td>( s = 2 )</td>
<td>( s = 3 )</td>
<td>( s = 4 )</td>
<td>( s = 5 )</td>
<td>( s = 6 )</td>
<td>( s = 7 )</td>
<td>( s = 8 )</td>
</tr>
<tr>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

### Part A: Write an inequality to represent the situation.

Coach De Leon purchased sports equipment: basketballs cost \( \$20 \) each, and soccer balls cost \( \$18 \) each. He had a budget of \( \$150 \). The graph shown below represents the number of basketballs and soccer balls he can buy given his budget constraint.

- \( b \): Number of basketballs
- \( s \): Number of soccer balls

The equation for the budget constraint is:

\[
18s + 20b = 150
\]
Let's Practice:

Find the solution sets to the systems of linear inequalities.

Problem:

1. Write a system of inequalities to represent the given situation.

Juan must purchase car insurance. He needs to earn at least $20 per hour. Each week is 8 hours because of football practice. Juan can earn $500 a week to cover the payments. The most he can work is 12 hours. He wants each week to cover the payments.

2. Graph the region of the solutions to the inequality.

3. Identify two different solutions for Juan's situation.

4. Interpret the graph.
Section 4: Linear Functions

**Part B:** Describe the system of inequalities that describes:

\[
\begin{array}{cccc}
(0,0) & (1,6) & (-2,7) & (-9,1) \\
(-5,10) & (-9,5) & (-8,3) & (-1,9) \\
(9,0) & (9,6) & (-3,8) & \\
(-1,9) & & & \\
\end{array}
\]

**Part A:** Underline the ordered pairs below that represent solutions to the system of inequalities.

**Try It!**

**2.** Anna is an avid reader. Her generous grandparents gave her money for her birthday, and she decided to spend at most $150.00 on books. Reading Spot is running a special: all paperback books are $8.00 and hardback books are $12.00. Anna wants to purchase at least 12 books. Anna is an avid reader. Her generous grandparents gave her money for her birthday, and she decided to spend at most $150.00 on books. Reading Spot is running a special: all paperback books are $8.00 and hardback books are $12.00. Anna wants to purchase at least 12 books.

a. Write a system of inequalities to represent the situation.

\[
\begin{align*}
&8p + 12h \leq 150 \\
&h \geq 12
\end{align*}
\]

b. Graph the region of the solutions to the inequalities.

c. Name two different solutions for Anna’s situation.
Section 5 – Quadratic Functions – Part 1
The following Mathematics Florida Standards will be
covered in this section:

Use the structure of an
MAFS.912.A-SSE.1.2
expression to identify ways to
rewrite it. For example, see
!" −$" as (!&)& − ($&)&, thus
recognizing it as a difference of
squares that can be factored
(!& − $&)(!& + $&).
MAFS.912.A-SSE.2.3.a.b Choose and produce an
equivalent form of an
expression to reveal and explain
properties of the quantity
represented by the expression.
Factor a quadratic
expression to reveal the
zeros of the function it
defines.

MAFS.912.F-IF.3.8.a

quadratic expression to
reveal the maximum or
minimum value of the
function it defines.
Write a function defined by an
expression in different but
equivalent forms to reveal and
explain different properties of
the function.
the process of factoring
and completing the square
in quadratic function to
show zeros, extreme values,
and symmetry of the graph,
and interpret these in terms
of a context.

MAFS.912.F-IF.2.4

MAFS.912.A-REI.2.4

For a function that models a
relationship between two
quantities, interpret key features
of graphs and tables in terms of
the quantities and sketch
graphs showing key features
given a verbal description of
the relationship. Key features
include: intercepts; intervals
where the function is increasing,
decreasing, positive, or
negative; relative maximums
! symmetries; end
and minimums;
behavior; and periodicity.

Solve quadratic equations in
one variable.
se the method of
completing the square to
transform any quadratic
equation in ! into an
equation of the form
(! − )&
that has the
same solutions. Derive the
quadratic formula from this
form.
lve quadratic equations
by inspection (e.g., for
!&
), taking square
roots, completing the
square, the quadratic
formula, and factoring, as
appropriate to the initial
form of the equation.
Recognize when the
quadratic formula gives
complex solutions.

4FDUJPO2VBESBUJD'VODUJPOTo1BSU

12


We can represent the scenario on a graph:

<table>
<thead>
<tr>
<th>Distance Traveled (in miles)</th>
<th>Time (in hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>160</td>
<td>4</td>
</tr>
<tr>
<td>120</td>
<td>3</td>
</tr>
<tr>
<td>80</td>
<td>2</td>
</tr>
<tr>
<td>40</td>
<td>1</td>
</tr>
</tbody>
</table>

Let’s revise linear functions.

Real-world Examples of Quadratic Functions

Section 5 - Topic 1
We can represent the distance traveled, in terms of time, hours, with the equation \( d(t) = 4t \). Linear functions always have a constant rate of change. In this section, we are going to discover a type of non-linear function. Quadratic functions always have a constant rate of change. In this section, we are going to discover a type of non-linear function. Liam then used his calculator to find the equation of the function:

\[ h(t) = -16t^2 + 300 \]

Liam then recorded his observations in the following table.

<table>
<thead>
<tr>
<th>Time (in seconds)</th>
<th>Height (in feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>300</td>
</tr>
<tr>
<td>0.6</td>
<td>283.9</td>
</tr>
<tr>
<td>1</td>
<td>235.1</td>
</tr>
<tr>
<td>1.4</td>
<td>188.4</td>
</tr>
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<td>2</td>
<td>144</td>
</tr>
<tr>
<td>3</td>
<td>42.4</td>
</tr>
</tbody>
</table>

Why do you think that the rate of change is not constant?

What is the independent variable?

What is the dependent variable?

What do you notice about the rate of change?

What do you notice about the rate of change?

Consider the following situation:

Liam dropped a watermelon from the top of a 300 ft tall building. He wanted to know if the watermelon was falling at a constant rate over time. He filmed the watermelon's fall and calculated the rate of change. He then recorded his observations in the following table.

Liam then recorded his observations in the following table.

<table>
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</tr>
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</table>

Why do you think that the rate of change is not constant?

What is the independent variable?

What is the dependent variable?

What do you notice about the rate of change?
In Liam’s graph, what was the watermelon’s height when it hit the ground?

The time when the watermelon’s height was at zero is called the solution to this quadratic equation. We also call these the ________________ of the equation.

There was only one solution to Liam’s equation. Describe a situation where there could be two solutions.

What about no solutions?

To solve a quadratic equation using a graph:

Look for the ________________ of the graph.

The solutions are the values where the graph intersects the ________________.

Zeros = x-intercepts = Solutions

This graph is called a ________________.

The general form of the equation is ________________.

We call this a non-linear function a ________________.

Important facts:

---

If you need any further clarification or additional assistance, feel free to ask!
The maximum or minimum point of the parabola is called the vertex.

1. What are the solutions to the quadratic equation graphed below?

2. Aaron shoots a water bottle rocket from the ground. A graph of height over time is shown below. According to the graph, how many seconds did the rocket hit the ground? (Round to the nearest tenth.)

3. What type of function best models the rocket's motion?

b. After how many seconds did the rocket hit the ground?

c. Estimate the maximum height of the rocket.
Section 5: Quadratic Functions - Part I

10x^3 - 14x^2 + 12x

Distributive Property:

| 12x | -14x^2 | 10x^3 |

Area Model:

common factor of an expression.
We can use these some methods to factor out the greatest

3(x + 2y - 7z)

Distributive Property:

Area Model:

polynomials.
Let's review the two methods we used for multiplying

Factoring Quadratic Expressions
Section 5 - Topic 2

Part D: What does the vertex represent?

Part C: Box the vertex of the graph.

Part B: What do the solutions represent?

Part A: Circle the solutions to the quadratic function

Selling Price

Profit

Electronic Sales

Part B: Graphed above.

1. Jordan owns an electronics business. During her first year in
Use the distributive property to write an equivalent expression for 
\((x + 3)(x + 5)\).

Use the area model to write an equivalent expression for 
\((x + 5)(x + 3)\).

Consider the resulting binomial: \(2x^2 + 11x + 15\).

Let’s consider how we can use this and the distributive property to factor a quadratic expression.

What are the two middle terms of the expanded form?

The first term of the binomial can always be found in the first rectangle.
The second term of the binomial is the product of the first and last term of the trinomial.
The product of the two middle terms of expanded form are equal to the product of the first and last term of the trinomial.

Notice that the product of the two middle terms of expanded form also sum to the middle term of the trinomial.

We can also use the distributive property to factor a quadratic expression.

Let’s consider how we can use this and the distributive property to factor a quadratic expression.

We can use this same area model to factor a quadratic expression.
Let's Practice!

1. Consider the quadratic expression $x^2 + 4x - 4$.

a. Factor using the area model.

b. Factor using the distributive property.

c. Factor the polynomial by grouping the first 2 terms and the last 2 terms.

Tip: The factors should always result in the original using the distributive property. The product of the factors should equal to the original trinomial. You can check your answer by factoring by every factor by multiplying the factors.
Try It!

2. Consider the quadratic expression $4w^2 - 21w + 20$.

b. Factor using the distributive property.

c. Factor using the area model.

1. Identify all factors of the expression $18x^2 - 9x - 5$.

- $3x + 1$
- $3x + 5$
- $18x - 5$
- $6x - 5$
- $5x + 5$
Solving Quadratics by Factoring

Solving a quadratic equation by factoring:

To solve a quadratic equation by factoring:

Step 1: Set the equation equal to zero.
Step 2: Factor the quadratic.
Step 3: Set each factor equal to zero and solve.
Step 4: Write the solution set.

The zero product property states that if the product of two factors is zero, then one (or both) of the factors must be zero. Once a quadratic equation is factored, we can use the zero product property to solve the equation.

Let's Practice!

1. Solve for $x$ by factoring $x^2 + 17x + 3 = 0$.

2. Solve for $x$ by factoring $y^2 + 8y + 15 = 0$.
Tyra solved the quadratic equation \( 4x^2 - 4x - 6 = 0 \) by factoring. Her work is shown below:

1. Tyra solved the quadratic equation \( x^2 - 10x - 24 = 0 \) by:

\[
\begin{align*}
\text{Step 1:} & \\
\text{Step 2:} & \\
\text{Step 3:} & \\
\text{Step 4:} & \\
\text{Step 5:} & \\
\text{Step 6:} & \\
\text{Step 7:} & \\
\text{Step 8:} & 
\end{align*}
\]

Tyra did not find the correct solutions. Investigate the steps, decipher her mistakes, and explain how to correct Tyra’s work.
Section 5: Quadratic Functions – Part 1

Solving Other Quadratics by Factoring

Many quadratic equations will not be in standard form:

The equation won’t always equal zero.

There may be a greatest common factor (GCF) within all of the terms.

The equation won’t always equal zero.

Many quadratic equations will not be in standard form.

Let’s Practice!

1. Solve for \( d \): \( d^2 + 36 = 13p \)

2. Solve for \( x \): \((x + 4)(x - 5) = -8\)

3. Solve for \( m \): \( 3m^2 + 30m - 168 = 0 \)

4. Solve for \( m \): \( 6d^2 + 5d = 1 \)

Try it!
There are a few special cases when solving quadratics by factoring.

**Perfect Square Trinomials**

A quadratic expression can be factored as a perfect square trinomial if it is of the form $a^2 + 2ab + b^2$.

**Recognizing a Perfect Square Trinomial**

- If $a^2 + 2ab + b^2$ is a perfect square trinomial, then
  $$(a + b)^2 = a^2 + 2ab + b^2$$

- If $a^2 - 2ab + b^2$ is a perfect square trinomial, then
  $$(a - b)^2 = a^2 - 2ab + b^2$$

**Factoring a Perfect Square Trinomial**

$$a^2 - 2ab + b^2 = (a - b)^2$$
$$a^2 + 2ab + b^2 = (a + b)^2$$
1. Solve for $x$: $4x^2 + 49 = -28x$

2. Solve for $b$: $b^2 - 10b + 25 = 0$

3. Determine whether $16x^2 + 8x + 121$ is a perfect square trinomial. Justify your answer.

4. Determine whether $x^2 - 8x + 64$ is a perfect square trinomial. Justify your answer.

5. What do you notice about the number of solutions to perfect square quadratic equations?

6. Sketch the graph of a quadratic equation that is a perfect square trinomial.

---

Let's Practice!

1. Determine whether $x^2 + 14x + 49$ is a perfect square trinomial. Justify your answer.
Difference of Squares

Use the distributive property to multiply the following binomials.

1. \((a + b)(a - b)\)

2. \((x + 5)(x - 5)\)

Describe any patterns you notice.

8. Solve the equation \(0 = 121p^2 - 100\).

Try It!

7. Solve the equation \((5x + 3)(5x - 3) = 49x^2 = 64\) by factoring.

Let's Practice!
Which of the following expressions are equivalent to $-\frac{t}{u}$? Select all that apply.

1. $\left(\frac{t}{u}\right)^{-1}$
2. $\left(\frac{t}{u}\right)^{-2}$
3. $\left(\frac{u}{t}\right)^{-1}$
4. $\left(\frac{u}{t}\right)^{-2}$

---

Section 5: Quadratic Functions - Part I

Solving Quadratics by Taking Square Roots

Consider the following quadratic equation.

$2x^2 - 36 = 0$

Solve for $x$ by taking the square root.

Step 1: Get the variable on the left and the constant on the right.

Step 2: Then take the square root of both sides of the equation. (Don't forget the negative root!)

Solve for $x$ by taking the square root.

$2x^2 - 36 = 0$
Let's Practice!

1. Solve $x^2 - 5x + 80 = 0$.

Try It!

2. Solve $-2x^2 + 17 = 179$.

Beat the Test!

1. What is the smallest solution to the equation $2x^2 + 17 = 179$?

2. A rescuer on a helicopter that is 50 feet above the sea drops a lifebelt. The distance from the lifebelt to the sea, in feet, can be modeled by the equation $h(t) = -16t^2 + s$, where $h(t)$ represents the lifebelt's height from the sea at any given time, $t$ is the time in seconds, and $s$ is the initial height from the sea. How long will it take for the lifebelt to reach the sea?

Round your answer to the nearest tenth of a second.
Recall how we factored perfect square trinomials.

If \( x^2 + 2ax + a^2 \) is a perfect square trinomial, then \( x^2 + 2ax + a^2 = (x + a)^2 \) and \( a^2 - 4a + 4 \) is a perfect square trinomial. Then recall how we factored perfect square trinomials.

Step 1: Write the equation in standard form.

Step 2: Move the constant term to the right side of the equation.

Step 3: If the coefficient of the \( x^2 \) term does not equal 1, then factor out the coefficient.

Step 4: Divide the coefficient of the middle term by two and square the result. Use the addition property of equality to make the trinomial a perfect square.

Solve \( x^2 + 22x - 71 \) by completing the square.

Let's see how this can be used to solve quadratic equations.

Let's start by determining what number \( a \) we can add to a quadratic expression to make it a perfect square trinomial.

What value could be added to the quadratic to make it a perfect square trinomial?

Sometimes you won't be able to solve a quadratic equation so that you can complete the square to factor and solve. However, you can rewrite the quadratic by factoring.
Section 5: Quadratic Functions - Part I

Step 5: Factor and solve the perfect square trinomial.

Let's Practice!

1. Complete the square to solve $3x^2 + 12x = -31$

Try It!

2. Complete the square to solve $2x^2 - 8x = -5$

<table>
<thead>
<tr>
<th>A. $2(x + 6)^2 = 29 + 72$</th>
<th>B. $x + 6 = \pm\sqrt{50.5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C. $2(x + 12x + 36) = 29 + 72$</td>
<td>D. $x = -6 \pm \sqrt{50.5}$</td>
</tr>
<tr>
<td>E. $x + 6 = \sqrt{50.5}$</td>
<td></td>
</tr>
</tbody>
</table>

Demonstrate how to solve $2x^2 + 24x = 0$ by completing the square. Place the equations in the correct order.

Beat the Test!
Factor the trinomial.

Subtract the constant term from both sides.

Take the square root of both sides.

Factor the trinomial.

Use the addition property of equality to write a perfect square trinomial.

Divide the coefficient of the $x^2$ term by two and square the result. Determine what you should add to create a perfect square trinomial.

Consider the equation $ax^2 + bx + c = 0$, where $a \neq 0$. Recall our steps for completing the square as a method for solving for $x$.

We can use the process of completing the square to derive a formula to solve any quadratic equation.

Section 5 – Topic 8
Deriving the Quadratic Formula
Section 5: Quadratic Functions - Part 1

Solving Quadratics Using the Quadratic Formula

For any quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$.

**Step 1:** Set the quadratic equation equal to zero.

To use the quadratic formula:

$\frac{\sqrt{b^2 - 4ac}}{2a} = x$

**Step 2:** Identify $a$, $b$, and $c$.

**Step 3:** Substitute $a$, $b$, and $c$ into the quadratic formula and evaluate to find the zeros.

For any quadratic equation $ax^2 + bx + c = 0$, the quadratic formula is:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Complete the missing steps in the derivation of the quadratic formula.
1. Use the quadratic formula to solve $x^2 - 4x + 3 = 0$. Consider the graph of the quadratic equation.

2. Use the graph of the quadratic equation.

3. Use the quadratic formula to solve $2w^2 + w = 5$.

4. Use the quadratic formula to solve $3y^2 - 11 = 204$.

Try III.
What is the maximum height?

From what height was the object launched?

What is the maximum height the object reached?

What is the height versus time equation?

Consider an object being launched into the air. We compare quadratic functions.

Let's consider solving real-world situations that involve quadratic functions.

1. Your neighbor's garden measures 12 meters by 16 meters.

Part A: Write the quadratic formula to solve for the width of the pathway.

Part B: Use the quadratic formula to find the width, w, of the pathway.
Question

How to Answer it

1. From what height was the object launched?

This is the $-c$-intercept. In the standard form, $y = ax^2 + bx + c$, $-c$ is the $y$-intercept.

2. How long did it take the object to reach its maximum height?

This is the $x$-coordinate of the vertex. The $x$-intercepts are the solution(s), or zero(s), of the quadratic function. $x = -b + c\sqrt{b^2 - 4ac}$

3. What was the maximum height?

This is the $y$-coordinate of the vertex. Substitute the $x$-coordinate from the step above and evaluate to find $y$. In vertex form, the height is $y$ and the vertex is $(x, y)$.

4. At what time did the object on the graph, look at the $x$-intercepts(s). The $x$-intercepts are the solution(s), or zero(s), of the quadratic function. $x = -b + c\sqrt{b^2 - 4ac}$

5. At what time(s) was the object on the ground?

The $x$-intercepts are the solution(s), or zero(s), of the quadratic function. Solve by factoring, using the quadratic formula, or by completing the square. The $x$-intercept(s) is the $x$-coordinate of the vertex of symmetry. The $y$-intercept is the vertex for $y = ax^2 + bx + c$.

a. When does the ball reach its maximum height?

b. What is the maximum height of the ball?

Let's Practice!!

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Answer It</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. From what height was the object launched?</td>
<td>This is the $-c$-intercept. In the standard form, $y = ax^2 + bx + c$, $-c$ is the $y$-intercept.</td>
</tr>
<tr>
<td>2. How long did it take the object to reach its maximum height?</td>
<td>This is the $x$-coordinate of the vertex. The $x$-intercepts are the solution(s), or zero(s), of the quadratic function. $x = -b + c\sqrt{b^2 - 4ac}$</td>
</tr>
<tr>
<td>3. What was the maximum height?</td>
<td>This is the $y$-coordinate of the vertex. Substitute the $x$-coordinate from the step above and evaluate to find $y$. In vertex form, the height is $y$ and the vertex is $(x, y)$.</td>
</tr>
<tr>
<td>4. At what time did the object on the graph, look at the $x$-intercepts(s).</td>
<td>The $x$-intercepts are the solution(s), or zero(s), of the quadratic function. Solve by factoring, using the quadratic formula, or by completing the square. The $x$-intercept(s) is the $x$-coordinate of the vertex of symmetry. The $y$-intercept is the vertex for $y = ax^2 + bx + c$.</td>
</tr>
<tr>
<td>5. At what time(s) was the object on the ground?</td>
<td>The $x$-intercepts are the solution(s), or zero(s), of the quadratic function. Solve by factoring, using the quadratic formula, or by completing the square. The $x$-intercept(s) is the $x$-coordinate of the vertex of symmetry. The $y$-intercept is the vertex for $y = ax^2 + bx + c$.</td>
</tr>
</tbody>
</table>
A neighborhood is throwing a fireworks celebration for the 4th of July. A bottle rocket was launched upward from the ground with an initial velocity of 66 feet per second. The formula for vertical motion of an object is 

\[ h(t) = -16t^2 + vt + s \]

where \( h \) is the gravitational constant, \( v \) is the initial velocity, \( s \) is the initial height, and \( t \) is the time elapsed in seconds.

Part A: What function describes the height, \( h \), of the bottle rocket after \( t \) seconds have elapsed?

Part B: What is the maximum height of the bottle rocket?

Part C: What is the height of the bottle rocket after 2 seconds?

Part D: When is the ball 65 feet in the air? Explain.

Part E: How long does it take until the golf ball hits the ground?

Try It!
### Section 6 – Quadratic Functions – Part 2

The following Mathematics Florida Standards will be covered in this section:

- **MAFS.912.A.CED.1.2**: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

- **MAFS.912.F.IF.2.4**: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

- **MAFS.912.F.IF.3.7.a**: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

- **MAFS.912.F.IF.3.8.a**: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

This section focuses on quadratic functions.

- **MAFS.912.A.REI.2.4.b**: Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, using the quadratic formula, completing the square, and recognizing when the quadratic formula gives complex solutions.

- **MAFS.912.A.REI.2.4.c**: Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, using the quadratic formula, completing the square, and recognizing when the quadratic formula gives complex solutions.

### Using Technology

This section will use technology to help solve and graph quadratic functions.

**Additional Resources**

- **MAPS.912.F.IP.3.8.a**: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- **MAPS.912.F.IP.3.7.a**: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

- **MAPS.912.A.REI.2.4.b**: Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, using the quadratic formula, completing the square, and recognizing when the quadratic formula gives complex solutions.

- **MAPS.912.A.REI.2.4.c**: Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, using the quadratic formula, completing the square, and recognizing when the quadratic formula gives complex solutions.

- **MAPS.912.A.REI.2.4.d**: Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions.

**Derived from Quadratic Functions**

- **MAPS.912.A.REI.2.4.e**: Derive the quadratic formula from this form.

- **MAPS.912.A.REI.2.4.f**: Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers $a$ and $b$.

- **MAPS.912.A.REI.2.4.g**: Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, using the quadratic formula, completing the square, and recognizing when the quadratic formula gives complex solutions.

**In this section:**

- **MAPS.912.A.CED.1.2**: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

- **MAPS.912.F.IP.3.9**: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
Explain why the coordinates of the points where the graphs of the equations \( y = mx + c \) and \( y = nx + d \) intersect are the solutions of the equation \( mx + c = nx + d \); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where \( m \) and/or \( n \) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
Let's review some things we learned earlier about the observations from a graph of a quadratic function.

1. The graph shows the height of a rocket from the time it was launched from the ground. Use the graph to answer the questions below.

a. What is the y-intercept?

b. What does the y-intercept represent?

The graph is labeled with the time in seconds on the x-axis and the height in feet on the y-axis. The graph shows a parabolic shape with a vertex at a certain point.

Let's focus on the graph:

- **Y-intercept:**

- **X-intercept(s):**

- **Axis of symmetry:**

- **Vertex:**
Step 1: Substitute the vertex \((h, k)\) and a point on the graph.

Step 2: Solve for \(a\).

\[ f(x) = a(x - h)^2 + k. \]

Step 3: Substitute another point on the graph \((x', y')\), and the coordinates of the vertex \((h, k)\) into the standard form of a quadratic function:

\[ \text{Vertex Form: } f(x) = a(x - h)^2 + k. \]

There is another form of the quadratic equation called the vertex form.

\[ f(x) = ax^2 + bx + c. \]

Recall the standard form of a quadratic equation:

\[ ax^2 + bx + c. \]

We can also use the graph to write the equation of the graph.

c. What are the \(x\)-intercepts of the graph?

d. What do the \(x\)-intercepts represent?

e. What is the maximum height of the rocket?

f. When will the rocket reach its maximum height?

g. When is the graph increasing?

h. When is the graph decreasing?

i. What is the domain of the graph?

j. What is the range of the graph?
Section 6: Quadratic Functions - Part 2

3. Consider the graph below.

b. Write the equation of the graph.
a. State five observations about the graph.

b. Write the function for the graph in vertex form.
a. Substitute the vertex, $(h, k)$, and the coordinates of another point on the graph, $(x, f(x))$, into $f(x) = a(x - h)^2 + k$ and solve for $a$.

2. Recall our graph from exercise #1.
Use the quadratic formula to find the zeros of the function.

**Where does the parabola intersect the x-axis?**

Consider the graph of the function $f(x) = x^2 - 4x + 4$.

Let's use the quadratic formula to discuss the nature of the solutions.

**Nature of the Solutions of Quadratics**

Section 6 – Topic 2

The graph represents the function $f(x) = -(x - 1)^2 + 9$.

- The graph has zeros at $-4$ and $2$.
- The graph's line of symmetry is the x-axis.
- The graph has an x-intercept at $(2, 0)$.
- The graph has a maximum point at $(-1, 4)$.
- The graph has a y-intercept at $(0, 9)$.

Which statements about this graph are true? Select all that apply.

1. The graph of a quadratic function is shown below.

**Beat the Test!**
Consider the graph of the function \( f(x) = x^2 + 6x + 8 \).

Consider the graph of the function \( f(x) = x^2 + 6x - 11 \).

Where does the parabola intersect the x-axis?

Use the quadratic formula to find the zero(s) of the function.

Where does the parabola intersect the x-axis?

Use the quadratic formula to find the zero(s) of the function.
Let's Practice!

1. Use the discriminant to determine if the following quadratic equations have complex or real solution(s).
   a. \( f(x) = x^2 - 8x + 20 \)
   b. \( x^2 - 6x + 9 = 0 \)
   c. \( 2x^2 - 3x - 10 = 0 \)

When using the quadratic formula, if the discriminant (part under the radical) results in a negative number, then the solutions are non-real, complex solutions.

Try It!

2. Create a quadratic equation that has complex solutions.
   Justify your answer.

3. Create a quadratic equation that has one real solution.

Let's Practice!

The solutions are non-real, complex solutions.

When using the quadratic formula, if the discriminant (part under the radical) results in a negative number, then the discriminant of the quadratic (part under the radical) is less than zero, indicating that the solutions are complex.
Graph function \( H(t) \) on the following coordinate grid.

Complete the table below.

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>Elevation (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.25</td>
<td>0.5</td>
</tr>
<tr>
<td>0.75</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1.25</td>
</tr>
</tbody>
</table>

Function \( H(t) = 25 - 16t^2 \), where \( t \) is time in seconds.

Suppose you jump into a deep pool of water from a diving platform that is 25 feet above the ground. Your height in feet is modeled by the equation \( H(t) = 25 - 16t^2 \). Which of the following quadratic equations have real solutions? Select all that apply.

\[
\begin{align*}
8 - 2x &= x^2 \quad \square \\
25x^2 &= 3x + 10 \quad \square \\
0 &= 9 + x^2 - 2x \quad \square \\
0 &= 6 + x^2 - 2x \quad \square \\
11 &= 5x + 2x^2 \quad \square \\
\end{align*}
\]
A business owner recorded the following data for an entire year of sales.

<table>
<thead>
<tr>
<th>Month</th>
<th>Sales (thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>45</td>
</tr>
<tr>
<td>Feb</td>
<td>34</td>
</tr>
<tr>
<td>Mar</td>
<td>54</td>
</tr>
<tr>
<td>Apr</td>
<td>63</td>
</tr>
<tr>
<td>May</td>
<td>70</td>
</tr>
<tr>
<td>June</td>
<td>71</td>
</tr>
<tr>
<td>July</td>
<td>31</td>
</tr>
<tr>
<td>Aug</td>
<td>45</td>
</tr>
<tr>
<td>Sept</td>
<td>64</td>
</tr>
<tr>
<td>Oct</td>
<td>54</td>
</tr>
<tr>
<td>Nov</td>
<td>38</td>
</tr>
<tr>
<td>Dec</td>
<td>24</td>
</tr>
</tbody>
</table>

2. A construction company uses square-shaped lots of various sizes to build houses. The CEO of the company decided to diversify her lots and now has houses built on rectangular lots that are 6 feet longer and 4 feet narrower than the square lots. According to the CEO, the size of the square lots is relative to the size of the square lots.

b. Complete the table below and graph function.

c. What is the function that models the size of the square lots?
Section 6: Quadratic Functions – Part 2

1. Consider the following table of values:

<table>
<thead>
<tr>
<th>x</th>
<th>-70</th>
<th>-30</th>
<th>0</th>
<th>0</th>
<th>-16</th>
<th>-16</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>4</td>
<td>2</td>
<td>-1</td>
<td>-3</td>
<td>-4</td>
<td>-5</td>
</tr>
</tbody>
</table>

Which of the following is the graph corresponding to the table of values?

b. What type of business might be represented by this graph?

c. Would the quadratic model be an appropriate way to model data for this business going forward? Justify your answer.

a. Plot the data on the graph below.
Section 6: Graphing Quadratics

Topics:

4. Graphing Quadratics Using the Vertex and Intercepts

Given a quadratic equation in standard form, \( f(x) = ax^2 + bx + c \), if needed, use the axis of symmetry to find a reflection point. In the equation, find and plot the y-intercept, which is the constant c.

Step 4. Find and plot the y-intercept, which is the constant c.

Step 3. Find \( f(h) \), the y-coordinate of the vertex, by substituting \( h \) into the equation. Plot the vertex, \((h, f(h))\), on the graph.

Step 2. Find and graph the axis of symmetry using the formula \( x = \frac{-b}{2a} \). This is also the x-coordinate of the vertex.

Step 1. Use the a-value to determine if the graph should open upward (positive a) or downward (negative a).

Following Page:

Step 5. Find and plot the x-intercepts of the function.
Let's Practice

1. Consider the function \( f(x) = -x^2 + 4x + 21 \).

   a. Use the quadratic formula. Factoring is one option, but you can always use the quadratic formula. Find and plot the x-intercepts of the function.

   b. Find the axis of symmetry using the formula \( x = \frac{-b}{2a} \). This is also the x-coordinate of the vertex. Find the x-coordinate of the vertex, \( f(h) \), and plot it on the graph.

   c. Find the y-coordinate of the vertex by substituting \( h \) into the equation. Plot the vertex, \( (h, k) \), on the graph.

   d. Find and plot the y-intercept, which is the constant \( c \). If possible, use the axis of symmetry to find a reflection point.

   e. Find and plot the x-intercepts, which is the constant \( c \). If possible, use the axis of symmetry to find a reflection point.

   f. Use the \( a \)-value to determine if the graph should open upward (positive \( a \)) or downward (negative \( a \)).
Try It!

2. Jorah starts at the top of SlotZilla Zipline™ in Las Vegas and rides down Fremont Street. The equation \( h(t) = -23.4t^2 + 114 \) models Jorah's height, in feet, above the ground over time, \( t \) seconds, spent riding the zip line.

a. What is the vertex of the function \( h(t) \)?

b. When will Jorah reach the ground?

c. Sketch the graph that models Jorah's height over the time spent riding the zip line.
Section 6: Quadratic Functions - Part 2

Graphing Quadratics Using Vertex Form - Part 1

Vertex Form: $f(x) = a(x - h)^2 + k$

Let’s review vertex form.

Vertex Form Using Quadratic Functions

Section 6 - Topic 5

What is the vertex of the function?

For example, function $s(t) = -16 \left( \frac{t}{2} \right)^2 - 16$ models the height, in meters, of a ball that is launched upward from a platform of a residential building.

The vertex of a parabola is the point that is compressed or stretched.

Coefficient $a$ determines if the graph opens upward or downward.

Coefficient $a$ also determines if the parabola is vertically stretched or compressed when compared to $y = x^2$.

The vertex is the point $(h, k)$, where $h$ is the x-coordinate of the vertex and $k$ is the y-coordinate.

Does the graph of the function open upward or downward?

Identify the vertex:

The problem was marked incorrect:

On a test, Mia graphed the quadratic function:

$f(x) = x^2 - 10x - 24$. The vertex of the parabola is $(5, 49)$.
Let's Practice!

1. Graph the function $f(x) = (x - 3)^2 + 4$.

   To graph a quadratic in vertex form, follow these steps:
   
   a. Use the $a$-value to determine if the graph should open upward (positive $a$) or downward (negative $a$).
   b. Find and graph the vertex, $(h, k)$, and axis of symmetry, $x = h$.
   c. Find the $y$-intercept by substituting zero for $x$. Plot the $y$-intercept.
   d. Find the $x$-intercepts by substituting $x = 0$. Plot the $x$-intercepts.
   e. Use the key features to sketch the graph.
Section 6 - Topic 6

Graphing Quadratics Using Vertex Form - Part 2

Many times quadratic equations are not written in vertex form. You can always use the process of completing the square to rewrite them in vertex form. You can also use the quadratic formula to find the x-intercepts. Remember to solve for x using square roots. Plug the

Let's Practice!

1. Write the function \( f(x) = x^2 - 4x - 2 \) in vertex form. Then, graph the function.

   a. Write the function in standard form.

   b. Group the quadratic and linear term together.

   c. If \( a \) does not equal one, factor \( a \) out of the equation.

   d. Complete the square.

   e. Use the key features to sketch the graph.
Section 6: Quadratic Functions – Part 2

2. Write the function $q(x) = 2x^2 - 12x + 17$ in vertex form.

Try III

4. Determine the key features and graph the quadratic function on the coordinate plane below.

Then, graph the function.
Velma rewrote a quadratic function in vertex form.

Step 1:

Step 2:

Step 3:

Velma said that the vertex is (-1, 1). Is Velma correct? If not, identify the step in which Velma made the mistake and correct her work.

The graph of \( g(x) \) is shown below.
1. Complete the table to explore what happens when we add a constant to $f(x)$.

<table>
<thead>
<tr>
<th>$y$</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

$z - (x)f = (x)\eta$

$z + (x)f = (x)\delta$

$(x)f = (x)d$

$(x)f/\delta = (x)\eta$

$(x)fz = (x)\mu$

$z = (x)f = (x)\eta$

$z + (x)f = (x)\delta$

2. Sketch the graphs of each function on the same coordinate plane with the graph of $f(x)$.

Consider the following transformations on the dependent variable $f(x)$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

Consider the graph and table for the function $f(x)$.

### Functions

Transformations of the Dependent Variable of Quadratic Functions
Section 6: Quadratic Functions - Part 2

1. Given the function \( f(x) = x^2 + 3 \), identify the effect on the graph by replacing \( f(x) \) with the following:

- A. Vertically compressed
- B. Shifted \( f(x) \) down | \( k \) units.
- C. Reflected \( f(x) \) about the \( x \)-axis.
- D. Vertically stretched
- E. Shifted \( f(x) \) up | \( k \) units.

2. Sketch the graphs of each function on the same coordinate plane with the graph of \( f(x) \).

3. Complete the table to determine what happens when we multiply \( f(x) \) by a constant.

<table>
<thead>
<tr>
<th>( (x)f = (x)d )</th>
<th>( (x)f^\frac{2}{1} )</th>
<th>( (x)f = (x)u )</th>
<th>( (x)f = (x)w )</th>
<th>( (x)f )</th>
<th>( x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
<td>4</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
</tbody>
</table>

4. Sketch the graphs of each function on the same coordinate plane with the graph of \( f(x) \).
Consider the following transformations on the independent variable $x$:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>-2</td>
</tr>
</tbody>
</table>

Consider the graph and table for the function $f(x)$.
Section 6: Quadratic Functions - Part 2

1. Complete the table to determine what happens when you add a negative constant to \( f(x) \).

<table>
<thead>
<tr>
<th>( f(x) )</th>
<th>( f(x) = (x+2) )</th>
<th>( f(x) = (x-2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>(x)( \theta )</td>
<td>(x)( \theta )</td>
<td>(x)( \theta )</td>
</tr>
<tr>
<td>(x)( = (x+2) )</td>
<td>(x)( = (x-2) )</td>
<td>(x)( = (x+2) )</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

2. Sketch the graphs of \( f(x) \) and \( g(x) \) on the same coordinate plane.

3. Complete the table to determine what happens when you add a positive constant to \( f(x) \).

<table>
<thead>
<tr>
<th>( f(x) )</th>
<th>( f(x) = (x+2) )</th>
<th>( f(x) = (x-2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>(x)( \theta )</td>
<td>(x)( \theta )</td>
<td>(x)( \theta )</td>
</tr>
<tr>
<td>(x)( = (x+2) )</td>
<td>(x)( = (x-2) )</td>
<td>(x)( = (x+2) )</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

4. Sketch the graphs of \( f(x) \) and \( g(x) \) on the same coordinate plane.
Section 6: Quadratic Functions - Part 2

5. Complete the table to determine what happens when you multiply $x$ by a number greater than 1.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$x^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>$\frac{3}{2}$</td>
<td>$\frac{9}{4}$</td>
</tr>
</tbody>
</table>

Try III

8. Sketch the graph of $f(x)$ on the same coordinate plane with the graph of $g(x)$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$x^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>$\frac{3}{2}$</td>
<td>$\frac{9}{4}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x$</th>
<th>$x^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>$\frac{3}{2}$</td>
<td>$\frac{9}{4}$</td>
</tr>
</tbody>
</table>

6. Sketch the graph of $m(x)$ on the same coordinate plane with the graph of $f(x)$.
The table that represents the quadratic function \( f(x) \) is shown below.

\[
\begin{array}{ccc}
\hline
x & f(x) & \theta(x) \\
\hline
1 & 2 & 1 \frac{1}{2} \\
2 & 3 & 2 \\
3 & 4 & 3 \\
4 & 5 & 4 \frac{1}{2} \\
5 & 6 & 5 \\
\hline
\end{array}
\]

The graph of the system is shown below:

\[
9 + x^2 = \theta(x)
\]
\[
9 + x^5 + \frac{1}{2}x = \theta(x)
\]

Consider the following system of equations:


We can find solutions to systems of linear and quadratic equations by looking at a graph or table.

The table that represents the quadratic function \( \theta(x) \) is shown below.
We can also use a process called successive approximations. Consider the following system:

\[ \begin{align*}
4x + 2y &= x \\
3x + 2z &= x
\end{align*} \]

The table that represents the system is shown below.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>(z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>11</td>
<td>6</td>
</tr>
</tbody>
</table>

Find the absolute differences in \(x\) and \(y\) on the table above. In between which two \(x\) values must the positive solution lie? Which of the values does the solution lie closest to?

We can also identify the solutions by looking at tables. We can easily find the solutions by looking for the \(x\)-coordinate where \(x\) is a root of \(f\). The table that represents the system is shown below.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>(z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>11</td>
<td>6</td>
</tr>
</tbody>
</table>

Use the table to identify the solutions of \(x = (x)\).
Let's Practice!

1. Using the same system, complete the table below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>6.25</td>
<td>1.5</td>
</tr>
<tr>
<td>1.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.3</td>
<td>6.29</td>
<td>1.2</td>
</tr>
<tr>
<td>1.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.1</td>
<td>4.41</td>
<td>1.1</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(x)\theta$</td>
<td>$(x)f$</td>
<td>x</td>
</tr>
</tbody>
</table>

2. Find the absolute differences in $f(x)$ and $g(x)$ on the table above.

3. Use the table to find the positive solution (to the nearest tenth) for $f(x) = g(x)$.

4. The graphs of $f(x)$ and $g(x)$ are shown below.

Try III.
Section 6: Quadratic Functions – Part 2

Consider the following system of equations.

\[ y = x \]

The table below represents the system.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>6</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>4.5</td>
<td>2.25</td>
<td>3.5</td>
<td>0.75</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>5.5</td>
<td>2</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>2.5</td>
<td>6</td>
<td>2.5</td>
</tr>
<tr>
<td>6.5</td>
<td>3</td>
<td>6.5</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>7.5</td>
<td>5</td>
<td>7.5</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>8</td>
<td>6</td>
</tr>
</tbody>
</table>

Use successive approximations to find the negative solution for \( y = x \).

The table below represents the system.

\[ 8 + x = y \]
\[ 10 - 2x = (x \cdot \theta) \]

Consider the following system of equations.
The following Mathematics Florida Standards will be covered in this section:

- MAFS.912.F-LE.1.1: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

- MAFS.912.F-LE.1.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

- MAFS.912.F-LE.1.3: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. 

- MAFS.912.F-LE.1.4: Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

- MAFS.912.F-LE.1.5: Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

- MAFS.912.F-LE.1.6: Understand that the graph of an equation in two variables is the set of all solutions plotted in the coordinate plane, often forming a curve (which could be a line).
### Exponential Functions

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAFS.912.F-LE.2.5</td>
<td>Interpret the parameters in a linear or exponential function in terms of a context.</td>
</tr>
<tr>
<td>MAFS.912.F-IF.1.3</td>
<td>Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.</td>
</tr>
<tr>
<td>MAFS.912.F-IF.2.4</td>
<td>Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.</td>
</tr>
<tr>
<td>MAFS.912.F-IF.3.9</td>
<td>Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions.</td>
</tr>
<tr>
<td>MAFS.912.F-IF.3.8</td>
<td>Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).</td>
</tr>
<tr>
<td>MAFS.912.A-SSE.2.c</td>
<td>Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Use the properties of exponents to transform expressions for exponential functions.</td>
</tr>
<tr>
<td>MAFS.912.F-IF.2.6</td>
<td>Graph exponential functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude and using phase shift.</td>
</tr>
<tr>
<td>MAFS.912.F-IF.2.7.e</td>
<td>Sketch graphs showing key features of the function. In terms of the quantities, interpret key features of graphs and relates them to the context.</td>
</tr>
</tbody>
</table>

### Section 7: Exponential Functions
### Geometric Sequences

Consider the sequence $3, 6, 12, 24, \ldots$. What pattern do you notice in the sequence?

- **Geometric Sequences**
- **Section 7 – Topic 1**

#### Sequence in a Table:

Just like arithmetic sequences, we can represent this sequence in a table:

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Term</th>
<th>Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
<td>24</td>
</tr>
</tbody>
</table>

#### Function Notation

- $a_n$: a formula to find the $n$th term

#### Example:

Consider the sequence $3, 6, 12, 24, \ldots$. What pattern do you notice in the sequence?

### Graphs of Exponential Functions

- **Section 7 – Topic 5**
- **Part 1**

#### Exponential Functions

This is an example of an exponential function.

Each term in the sequence is the product of the previous term and some real number $r$. Each term in the sequence is the expression $a_n = r^n$.这就是几何序列的例子。

### Growth and Decay Rates of Exponential Functions

- **Topic 7: Graphs of Exponential Functions**

#### Exponential Functions

- **Topic 3: Exponential Functions**

### Comparing Linear, Quadratic, and Exponential Functions

- **Topic 9: Comparing Linear, Quadratic, and Exponential Functions**

#### Linear Functions

- **Part 1**

- **Part 2**

---

### Exponential Functions

- **Section 7 – Topic 5**
- **Part 1**

#### Growth and Decay Rates of Exponential Functions

- **Topic 6: Growth and Decay Rates of Exponential Functions**

#### Exponential Functions

- **Topic 3: Exponential Functions**

### Transformations of Exponential Functions

- **Topic 7: Transformations of Exponential Functions**

#### Exponential Functions

- **Topic 3: Exponential Functions**

### Real-World Examples of Arithmetic and Geometric Sequences

- **Topic 2: Real-World Examples of Arithmetic and Geometric Sequences**

---

### Geometric Sequences

- **Section 7 – Topic 1**

#### Function Notation

- $a_n$: a formula to find the $n$th term

---

### Geometric Sequences

- **Section 7 – Topic 1**

#### Function Notation

- $a_n$: a formula to find the $n$th term

---

### Geometric Sequences

- **Section 7 – Topic 1**

#### Function Notation

- $a_n$: a formula to find the $n$th term
### Section 7: Exponential Functions

We can use the recursive process, where we relate the previous term back to the first term. We can use the explicit process, where we relate the previous term of this sequence.

<table>
<thead>
<tr>
<th>Sequence</th>
<th>Term</th>
<th>Number</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(1)</td>
<td>1</td>
<td>1</td>
<td>A(1)</td>
</tr>
<tr>
<td>A(2)</td>
<td>2</td>
<td>2</td>
<td>A(2)</td>
</tr>
<tr>
<td>A(3)</td>
<td>3</td>
<td>1</td>
<td>A(3)</td>
</tr>
<tr>
<td>A(4)</td>
<td>4</td>
<td>1</td>
<td>A(3)</td>
</tr>
<tr>
<td>A(5)</td>
<td>5</td>
<td>1</td>
<td>A(4)</td>
</tr>
<tr>
<td>A(6)</td>
<td>6</td>
<td>1</td>
<td>A(5)</td>
</tr>
<tr>
<td>A(7)</td>
<td>7</td>
<td>1</td>
<td>A(6)</td>
</tr>
<tr>
<td>A(8)</td>
<td>8</td>
<td>1</td>
<td>A(7)</td>
</tr>
<tr>
<td>A(9)</td>
<td>9</td>
<td>1</td>
<td>A(8)</td>
</tr>
<tr>
<td>A(10)</td>
<td>10</td>
<td>1</td>
<td>A(9)</td>
</tr>
</tbody>
</table>

We can use the recursive formula that we could use to find any term in the sequence.

\[
A_n = A_1 \cdot r^{(n-1)}
\]

Where:
- \(A_n\) is the \(n\)th term of the sequence.
- \(A_1\) is the first term of the sequence.
- \(r\) is the common ratio.
- \(n\) is the term number.

To find the 10th term, we use the formula:

\[
A_{10} = A_1 \cdot r^{(10-1)} = A_1 \cdot r^9
\]

For the given sequence, where \(A_1 = 1\) and \(r = 2\), the 10th term can be calculated as:

\[
A_{10} = 1 \cdot 2^9 = 512
\]

Thus, the 10th term of this sequence is 512.
The recursive process uses the previous term, while the explicit process uses the first term.

The sequence can be found in the table:

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>48</td>
</tr>
<tr>
<td>6</td>
<td>96</td>
</tr>
</tbody>
</table>

Write an explicit formula that we could use to find any term in the sequence.
1. Consider the sequence $-1, 3, -9, 27, \ldots$.

   a. Write a recursive formula for the sequence.

   b. Write an explicit formula for the sequence.

   c. Find the $n$th term of the sequence.

2. The first four terms of a geometric sequence are 7, 14, 28, and 56.

   a. Write a recursive formula for the sequence.

   b. Write an explicit formula for the sequence.

   c. Find the 12th term of the sequence.

Section 7: Exponential Functions
An art gallery was showcasing a long photo of a geometric landscape. The picture was enlarged ten times, each time by \( \frac{3}{16} \) of the previous picture.

Enter formulas that will give the length of each enlarged print.

Recursive formula:

Explicit formula:

---

The founder of a popular social media website is trying to inspire gifted algebra students to study computer programming. He is offering two different incentive programs for students.

Option 1: Students will earn one penny for completing their first math, science, or computer-related college course. The amount earned will double for each additional course they complete.

Option 2: Students will earn one penny for completing their first math, science, or computer-related college course. For each subsequent course they complete, they will earn \( \frac{3}{2} \) more than the previous course.

Write a formula for each option.

Explicit formula:

Recursive formula:
Compare the two scholarship options in the tables below:

<table>
<thead>
<tr>
<th>Course</th>
<th>Option 1</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$2.400.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$2.380.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$2.200.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$2.140.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$2.000.01</td>
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<td></td>
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<td></td>
<td></td>
<td>$1.000.01</td>
</tr>
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<td></td>
<td></td>
<td>$0.900.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0.800.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0.700.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0.600.01</td>
</tr>
<tr>
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<td></td>
<td>$0.500.01</td>
</tr>
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<td>$0.400.01</td>
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<tr>
<td></td>
<td></td>
<td>$0.300.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0.200.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0.100.01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Course</th>
<th>Option 2</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$167.772.16</td>
</tr>
<tr>
<td></td>
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<td>$83.886.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$26.493.00</td>
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<td></td>
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<td>$22.000.00</td>
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<td>$31.000.00</td>
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<td></td>
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<td></td>
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<td></td>
<td></td>
<td>$41.908.19</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$50.642.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$60.500.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$70.700.00</td>
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<td></td>
<td>$80.000.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$90.000.00</td>
</tr>
</tbody>
</table>

**Pattern:**
- Each term is the sum of the previous term and 100.
- This arithmetic sequence follows.

**Option 2 is an arithmetic sequence.**

<table>
<thead>
<tr>
<th>Course</th>
<th>Option 2</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$167.772.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$83.886.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$26.493.00</td>
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<td></td>
<td></td>
<td>$22.000.00</td>
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<td></td>
<td></td>
<td>$31.000.00</td>
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<td></td>
<td></td>
<td>$22.222.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$32.323.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$41.908.19</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$50.642.50</td>
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<tr>
<td></td>
<td></td>
<td>$60.500.00</td>
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<tr>
<td></td>
<td></td>
<td>$70.700.00</td>
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<tr>
<td></td>
<td></td>
<td>$80.000.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$90.000.00</td>
</tr>
</tbody>
</table>

**Pattern:**
- Each term is the product of the previous term and two.
- This geometric sequence follows.

**Option 1 is a geometric sequence.**
Section 7: Exponential Functions

1. Consider the two scholarship options for studying computer science.
   a. Which scholarship option is better if your college degree requires math, engineering, or programming courses?
   b. What if your degree requires math, engineering, or programming courses?
   c. Do you think that these graphs represent discrete or continuous functions? Justify your answer.
   d. Do you think Option 1 would ever be offered as a scholarship? Why or why not?

2. Pablo and Lily are saving money for their senior trip next month. Pablo's goal is to save one penny on the first day of the month and triple the amount he saves each day for the next month. Pablo's goal is to save one penny on the first day of the month and triple the amount he saves each day for the next month. Pablo's savings plan is an example of a(n):
   a. geometric sequence
   b. arithmetic sequence
   c. arithmetic sequence
   d. geometric sequence

Lily's savings plan is an example of a(n):
   a. geometric sequence
   b. arithmetic sequence
   c. geometric sequence
   d. arithmetic sequence

a. Pablo's savings plan is an example of a(n):
   b. Pablo and Lily are saving money for their senior trip next month.
   c. Which person do you think will be able to meet their goal? Explain.
   d. What if your degree requires 25 math, engineering, or programming courses?
1. On Sunday, Chris and Caroline will begin their final preparations for a piano recital the following Saturday.

Caroline plans to practice 30 minutes on the Sunday prior to the recital. Chris plans to practice half Caroline’s time on Sunday, but will double his practice time every day leading up to the recital. Chris plans to practice 30 minutes on the Sunday prior to the recital and increase his practice time by 30 minutes every day leading up to the recital.

Part A: List Caroline and Chris’ practice times in the tables below.

<table>
<thead>
<tr>
<th>Days Since Sunday</th>
<th>Caroline’s Practice Time</th>
<th>Chris’ Practice Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Part B: Compare the graphs of Caroline and Chris’ exponential practice times. Identify each graph as linear or exponential.

Section 7: Exponential Functions
Section 7: Exponential Functions

1. Exponential Functions

Functions can be represented by:
- Verbal descriptions
- Algebraic equations
- Numeric tables
- Graphs

Let's review linear and quadratic functions.

**Exponential Functions**

**Algebraic equation:**

The situation is represented by the function $f(y) = 60y$. Your distance, $f(y)$, depends on your time, $y$, in hours.

$$f(y) = 60y$$

**Verbal description:**

You are driving to visit your best friend in Gulfport. Since you have a long drive ahead, you turn on your cruise control. The cruise control keeps your car traveling at a constant rate of 60 mph.

**Graph:**

![Graph of Exponential Function]

<table>
<thead>
<tr>
<th>Time (in hours)</th>
<th>Distance (in miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>120</td>
</tr>
<tr>
<td>3</td>
<td>180</td>
</tr>
<tr>
<td>4</td>
<td>240</td>
</tr>
<tr>
<td>5</td>
<td>300</td>
</tr>
<tr>
<td>6</td>
<td>360</td>
</tr>
<tr>
<td>7</td>
<td>420</td>
</tr>
</tbody>
</table>

**Numeric Table:**

<table>
<thead>
<tr>
<th>$y$</th>
<th>$f(y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>120</td>
</tr>
<tr>
<td>3</td>
<td>180</td>
</tr>
<tr>
<td>4</td>
<td>240</td>
</tr>
<tr>
<td>5</td>
<td>300</td>
</tr>
<tr>
<td>6</td>
<td>360</td>
</tr>
<tr>
<td>7</td>
<td>420</td>
</tr>
</tbody>
</table>
You are observing the height of a ball as it's dropped from a building. Because of gravity, the more time that passes, the faster the ball travels. The ball does not travel at a constant speed, like your car on cruise control.

You are performing an experiment in science class in which you start with 100 bacteria and the amount of bacteria doubles every hour.

### Graph:

![Graph showing quadratic function]

### Verbal description:

The height of the ball \( h(t) \) is a function of \( t \), or depends on \( t \), the time (in seconds).

The quadratic function can be represented by the equation \( h(t) = -16t^2 + 150 \).

### Numeric table:

<table>
<thead>
<tr>
<th>Time (in seconds)</th>
<th>Height (in feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>9.6</td>
<td>2</td>
</tr>
<tr>
<td>13.4</td>
<td>1</td>
</tr>
<tr>
<td>18.9</td>
<td>0.5</td>
</tr>
<tr>
<td>26.7</td>
<td>0</td>
</tr>
</tbody>
</table>

Section 7: Exponential Functions
Section 7: Exponential Functions

Algebraic equation:

1. Pick two points. It’s helpful to use the y-intercept and the coordinate where \( x = 3 \).
2. Substitute the coordinates into the exponential equation \( y = ab^x \). Solve for \( a \) and \( b \).
3. Substitute the coordinates into the exponential function.
4. Use the following steps to write the equation for the exponential function:
   - Substitute the values into the equation \( y = ab^x \).
   - Solve for \( a \) and \( b \).

<table>
<thead>
<tr>
<th>Time (in hours)</th>
<th>Amount of Bacteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
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<td>0</td>
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<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Numeric table: $p(x)$
Section 7: Exponential Functions

1. The table and graph below represent an exponential function.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
</tbody>
</table>

2. The table and graph below represent an exponential function.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>108</td>
<td>3</td>
</tr>
<tr>
<td>36</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
</tbody>
</table>

3. From a hypothesis relating the b term to one of the key features of the graph.

b. From a hypothesis relating the a term to one of the key features of the graph.

c. From a hypothesis relating the b term to one of the key features of the graph.

Try III

Did your earlier hypotheses hold true for this equation?
Section 7: Exponential Functions

1. Match the graphs below with the following functions:

- \( f(x) = a \cdot b^x \)
- \( f(x) = a \cdot b^{-x} \)
- \( f(x) = a \cdot 2^x \)
- \( f(x) = a \cdot 3^x \)

Which key feature of the exponential function does the \( b \) term represent?

Which key feature of the exponential function does the \( a \) term represent?

Exponential Functions

Let's review what we learned in the previous video about exponential functions.

Graphs of Exponential Functions – Part 4
1. Consider the exponential equation \( y = 2^x \).

a. Sketch the graph of the exponential equation.

b. Is the graph increasing or decreasing?

c. Describe the end behavior of the graph.

As \( x \) increases, \( y \) __________.

As \( x \) decreases, \( y \) __________.

2. Consider the exponential equation \( y = \left(\frac{3}{4}\right)^x \).

a. Sketch the graph of the exponential equation.

b. Is the graph increasing or decreasing?

c. Describe the end behavior of the graph.

As \( x \) increases, \( y \) __________.

As \( x \) decreases, \( y \) __________.

Remember, you can always write an exponential function such as \( f(x) = a \cdot b^x \) in the form \( f(x) = \log_a (b^x) \) by writing the understood \( \log \) in the front.
Section 7: Exponential Functions

3. Consider the exponential equation $y = e^{-x}$.

a. Sketch the graph of the exponential equation.

b. Is the graph increasing or decreasing?

c. Describe the end behavior of the graph.

As $x$ increases, $y$ ____________.

As $x$ decreases, $y$ ____________.

4. Consider the exponential equation $y = -2^{x}$.

a. Sketch the graph of the exponential equation.

b. Is the graph increasing or decreasing?

c. Describe the end behavior of the graph.

As $x$ increases, $y$ ____________.

As $x$ decreases, $y$ ____________.
Sometimes we can use the properties of exponents to easily sketch exponential functions.

How can we use the properties of exponents to sketch the graph of \( y = a \cdot b^x \)?

Make a hypothesis about the relationship between the \( y \)-intercept, common ratio, and end behavior of a graph. Use your hypothesis to complete the table below.

<table>
<thead>
<tr>
<th>( a )</th>
<th>( b )</th>
<th>End Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>( &gt; 1 )</td>
<td>( &gt; 1 )</td>
<td>Increasing</td>
</tr>
<tr>
<td>( &gt; 1 )</td>
<td>( &gt; 0 )</td>
<td>Decreasing</td>
</tr>
<tr>
<td>( &lt; 1 )</td>
<td>( &gt; 1 )</td>
<td>Decreasing</td>
</tr>
<tr>
<td>( &lt; 1 )</td>
<td>( &lt; 0 )</td>
<td>Increasing</td>
</tr>
</tbody>
</table>

If you get confused about end behavior, you can sketch the graph of \( y = a \cdot b^x \) and the key features to see the end behavior.
Section 7: Exponential Functions

1. Use the properties of exponents to sketch the graph of $y = 2^{x-3}$. 

2. Use the properties of exponents to sketch the graph of $y = 3^{-x}$. 

Let’s Practice.
Growth and Decay Rates of Exponential Functions

Consider an exponential function in the form $f(x) = a^x$.

Assume that $a$ is positive.

If $a > 1$, the function is increasing.

If $0 < a < 1$, the function is decreasing.

What are some examples of exponential growth?

What are some examples of exponential decay?

1. The graph that represents the function $f(x) = -3 \cdot 2^x$ has a y-intercept at $-3$.
   - The graph is increasing by a common ratio of $2$.
   - $x$ decreases.

2. Which of the following have the same graphic representation as the function $f(x) = 8 \cdot 2^x$? Select all that apply.
   - $a^x = (1 - 2)^x$
   - $a^x = 2^{x+1}$
   - $a^x = 2^x$
   - $a^x = 2^{x^2}$
   - $a^x = 2^{x^2} + 1$
   - $a^x = 2^x + a$

BEAT THE TEST!
Section 7: Exponential Functions

1. Consider the exponential function $f(x) = 3^x$, which models the amount of money in Tyler’s savings account, where $x$ represents the number of years since Tyler invested the money.

a. Is the money in the account growing or decaying?

b. What is the rate of growth or decay?

c. What does $3^x$ represent?

2. Consider the exponential function $f(x) = 2^{0.1x}$, which models the value of Robert’s car, where $x$ represents the number of years since he purchased the car.

a. Is the value of Robert’s car growing or decaying?

b. What is the rate of growth or decay?

c. What does $2^{0.1x}$ represent?

TIP: To find the decay rate, you must subtract $3$ from $1$. To find the growth rate, you subtract $1$ from $3$. You will see the rate of growth/decay expressed as a decimal or a percent.
4. Consider the exponential function \( f(x) = \frac{1}{2} \cdot 1.08^x \), which models the amount of money remaining in Lola's retirement fund, where \( x \) represents the number of years since Lola began withdrawing the money.

a. What is the rate of growth or decay?

b. What does \( \frac{1}{2} \cdot 1.08^x \) represent?

3. Consider the exponential function \( f(x) = 25'000 \cdot 0.88^x \), which models the amount of money invested in a bond fund, where \( x \) represents the number of years since the money was invested.

a. What is the rate of growth or decay?

b. What does \( 25'000 \cdot 0.88^x \) represent?
BEAT THE TEST!

1. The equation $y = 2\times 0.1^{x}$ models exponential growth. The rate of growth is $10\%$.

2. The function $f(x) = 350 \cdot 0.75^x$ models the amount of money remaining in Alicia's summer budget, where $x$ represents the number of weeks since summer began. Which of the following are true statements? Select all that apply.
   - The function models exponential decay.
   - $0.75$ represents the amount of money Alicia had in the budget at the beginning of summer.
   - The rate of decay is $25\%$.
   - Alicia spent $262.50 during the first week of summer.
   - At the end of the second week, Alicia will have less than $217.5$ in the budget.
Consider the following transformations of \( f(x) \):

1. \( f(x - 3) \)
2. \( f(x + 3) \)
3. \( f(x) - 3 \)
4. \( -(x - 3) \)

Write a function to represent each transformed function and describe the transformation.
Section 7: Exponential Functions

Transformed function: $f(x) + 3$

Description:

1. Describe how $k$ affects the graph of the function $f(x) = 3x$. Assume $k < 1$.

   a. $f(x + k)$

   b. $f(x) - k$

   c. $k(f(x))$

2. The function $g(x)$ represents an exponential function. The ordered pair (6, -3) lies on the graph of $g(x)$.

   a. The function $f(x) = g(x) + 5$. Name a point on the graph of $f(x)$.

   b. The function $h(x) = g(x) + 3$. Name a point on the graph of $h(x)$.
### Section 7: Exponential Functions

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z + (x)f = (x).a )</td>
<td>Each transformation, consider the function ( f(x) = x^3 ). Describe the graph of ( z + (x)f ).</td>
</tr>
<tr>
<td>( (x - z)f = (x).u )</td>
<td></td>
</tr>
<tr>
<td>( (x)fz = (x).w )</td>
<td></td>
</tr>
<tr>
<td>( y - (x)f = (x).b )</td>
<td></td>
</tr>
<tr>
<td>( 2 + (x)f = (x).b )</td>
<td></td>
</tr>
</tbody>
</table>

1. The following graph represents the function \( f(x) \).
2. Recall the graph of \( f(x) = x^2 \). Describe the graph of \( x - 3 \).
3. Describe the graph of \( x^3 + 2 \).
4. Write the exponential function for the graph.
Section 7: Exponential Functions

Comparing Linear, Quadratic, and Exponential Functions – Part 1

### Linear Functions

<table>
<thead>
<tr>
<th>Equation</th>
<th>Shape</th>
<th>Rate of Change</th>
<th>Number of ( x )-intercepts</th>
<th>Number of ( y )-intercepts</th>
<th>Number of vertices</th>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
</table>

Sketch the graphs of three linear functions that show all of the possible combinations above.

### Quadratic Functions

<table>
<thead>
<tr>
<th>Equation</th>
<th>Shape</th>
<th>Rate of Change</th>
<th>Number of ( x )-intercepts</th>
<th>Number of ( y )-intercepts</th>
<th>Number of vertices</th>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
</table>

Sketch the graphs of three quadratic functions that show all of the possible combinations above.

### Exponential Functions

<table>
<thead>
<tr>
<th>Equation</th>
<th>Shape</th>
<th>Rate of Change</th>
<th>Number of ( x )-intercepts</th>
<th>Number of ( y )-intercepts</th>
<th>Number of vertices</th>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
</table>

Sketch the graphs of three exponential functions that show all of the possible combinations above.
Consider the following tables that represent a linear and a quadratic function and find the differences.

<table>
<thead>
<tr>
<th>Linear Function</th>
<th>Quadratic Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>((x))</td>
<td>((x))</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Sketch the graphs of two exponential functions that show all of the possible combinations above.

<table>
<thead>
<tr>
<th>Exponential Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range:</td>
</tr>
<tr>
<td>Domain:</td>
</tr>
<tr>
<td>Number of Vertices:</td>
</tr>
<tr>
<td>Number of Y-Intercepts:</td>
</tr>
<tr>
<td>Number of X-Intercepts:</td>
</tr>
<tr>
<td>Rate of Change:</td>
</tr>
<tr>
<td>Shape:</td>
</tr>
<tr>
<td>Equation:</td>
</tr>
</tbody>
</table>
How can you distinguish a linear function from a quadratic function?

Consider the following table that represents an exponential function.

How can you determine if a function is exponential by looking at a table?

<table>
<thead>
<tr>
<th>Key Feature</th>
<th>Linear</th>
<th>Quadratic</th>
<th>Exponential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of change is constant</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>2nd differences, but not 1st are constant</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Graph has no x-intercept</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Graph has 0 vertex</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Domain is all real numbers</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Range is ${y</td>
<td>y \geq 0}$</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Range is ${y</td>
<td>y &lt; 0}$</td>
<td>○</td>
<td>○</td>
</tr>
</tbody>
</table>

Consider the following table that represents an exponential function.

How can you distinguish a linear function from a quadratic function?

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Let's practice!

## Functions - Part 2

### Comparing Linear, Quadratic, and Exponential Functions - Topic 9

1. Identify whether the following key features indicate a model could be linear, quadratic, or exponential.

<table>
<thead>
<tr>
<th>Key Feature</th>
<th>Linear</th>
<th>Quadratic</th>
<th>Exponential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of change is constant</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>2nd differences, but not 1st are constant</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Graph has no x-intercept</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Graph has 0 vertex</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Domain is all real numbers</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Range is ${y</td>
<td>y \geq 0}$</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Range is ${y</td>
<td>y &lt; 0}$</td>
<td>○</td>
<td>○</td>
</tr>
</tbody>
</table>

Consider the following table that represents an exponential function.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>
### Section 7: Exponential Functions

1. Identify whether the following real-world examples should be modeled by a linear, quadratic, or exponential function.

<table>
<thead>
<tr>
<th>Real-World Example</th>
<th>Linear</th>
<th>Quadratic</th>
<th>Exponential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jumping from a high dive</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Appreciating value of property</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A dead body decaying</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Running a race at a constant speed</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kicking a ball into the air</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The same price all day</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Selling fruit and vegetables at</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growing a culture of bacteria</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Complete the following table so that \( f(x) \) represents a linear function and \( g(x) \) represents an exponential function.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>162</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>54</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>18</td>
</tr>
<tr>
<td>18</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>( x )</td>
<td>( x )</td>
</tr>
</tbody>
</table>

3. Determine whether each table represents a linear, quadratic, or exponential function.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>15</td>
<td>37</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>19</td>
<td>9</td>
</tr>
<tr>
<td>13</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>( x )</td>
<td>( x )</td>
</tr>
</tbody>
</table>

### Exercise

- **Beat the Test!**

The image contains a page from a textbook section on exponential functions. The page includes a table with various real-world examples and a task to identify the type of function (linear, quadratic, or exponential) that models each example. The page also contains a table with rows and columns, each containing values, and the task is to determine whether each table represents a linear, quadratic, or exponential function.
Section 8: Polynomial Functions

The following Mathematics Florida Standards will be covered in this section:

MAFS.912.F-IF.1.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MAFS.912.F-IF.3.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

MAFS.912.A-APR.2.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

### Topics in this Section

**Topic 1:** Finding Zeros of Polynomial Functions of Higher Degrees

**Topic 2:** End Behavior of Graphs of Polynomials

**Topic 3:** Graphing Polynomial Functions of Higher Degrees

---

What are the zeros of \( f(x) \)?

Consider the following graph of \( f(x) \).

How do you determine if \( x \) is a solution or zero for \( f(x) \)?

How can you find zeros when given the equation of a polynomial function in factored form?

How can you find zeros when given the graph of a polynomial function of higher degrees?

---

**Finding Zeros of Polynomial Functions of Higher Degrees**

### Key Concepts:

- **Finding Zeros of Polynomial Functions of Higher Degrees**
- **End Behavior of Graphs of Polynomials**
- **Graphing Polynomial Functions of Higher Degrees**

The following Mathematics Florida Standards will be covered in this section:
Consider the following fourth degree polynomial function.

\[ f(x) = x^4 - 4x^3 + 5x^2 - x + 1 \]

1. Consider the following graph of \( f(x) \).

Find the range of the function \( f(x) \) for the given domain \( \{ -2, -1, 0, 1, 2 \} \).

Does the above domain contain zeros of \( f(x) \)? Justify your answer.

Consider the following third degree polynomial function.

\[ h(x) = -x^3 - 5x^2 + x - 3 \]

Find the zeros of the function \( h(x) \).

Consider the following fourth degree polynomial function.

\[ g(x) = x^4 - 4x^3 + 5x^2 - x + 1 \]

2. What are the zeros of \( g(x) \)?
Section 8: Polynomial Functions

What are the other zeros of \( h(x) \)?

1. Consider the graph of \( h(x) \).
2. Justify your answer.
3. Are any zeros of \( h(x) \) found in the above domain?
4. Find the range of \( h(x) \) given the domain \([-1, 3]\).
5. Consider the function \( h(x) = x^3 - 3x^2 + 2 \).

Beat the Test!
Section 8: Polynomial Functions

End Behavior of Graphs of Polynomials

Make observations about the end behavior of the following graphs.

- $x = \infty$
- $x = -\infty$
Section 8: Polynomial Functions

Let's Practice!

1. Consider the following graph of \( f(x) \).

2. Describe the end behavior of the function.

   a. Does the function \( f(x) \) have an even or odd degree?

   b. Is the leading coefficient of \( f(x) \) positive or negative?

   c. Use your observations to sketch the graphs and complete the table.

   

<table>
<thead>
<tr>
<th>Leading Coefficient</th>
<th>Degree of Polynomial</th>
<th>End Behavior of Polynomials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive</td>
<td>Even</td>
<td>( f(x) ) as ( x \to \infty ), ( f(x) ) as ( x \to -\infty )</td>
</tr>
<tr>
<td>Negative</td>
<td>Even</td>
<td>( f(x) ) as ( x \to \infty ), ( f(x) ) as ( x \to -\infty )</td>
</tr>
<tr>
<td>Positive</td>
<td>Odd</td>
<td>( f(x) ) as ( x \to \infty ), ( f(x) ) as ( x \to -\infty )</td>
</tr>
<tr>
<td>Negative</td>
<td>Odd</td>
<td>( f(x) ) as ( x \to \infty ), ( f(x) ) as ( x \to -\infty )</td>
</tr>
</tbody>
</table>

\( f(x) = 8x^2 + 5x - 9x \).
Section 8: Polynomial Functions

4. Describe the end behavior of the function below.

\[ f(x) = x^2 - 2x + 7 \]

a. Justify your answer.

b. Is the leading coefficient of \( f(x) \) positive or negative?

c. Does the function \( f(x) \) have an even or odd degree?

3. Consider the following graph of \( f(x) \).

1. Determine which of the following statements is true for the function \( f(x) = 3x^2 + 7x - 42x^2 \).
Section 8: Polynomial Functions

Let's Practice!

1. Sketch the graph of the following polynomial.

\[(s + x)(s + x)(2 - x) = (x)(x)\]

Use the end behavior and zeros to sketch the graph of \(g(x)\).

Find the zeros of \(g(x)\).

Describe the end behavior of the graph of \(g(x)\).

Consider the following function.

\[(2 - x)(3 - x)(1 + x)(1 - x) = (x)(x)(x)(x)\]
Section 8: Polynomial Functions

1. Match each equation with its corresponding graph.

$$(z + x)(z - x)(z + x)(z - x) = A.$$  

$$(z + x)(z - x)(z + x)(z - x) = B.$$  

$$(z + x)(z - x)(z + x)(z - x) = C.$$  

$$(z + x)(z - x)(z + x)(z - x) = D.$$  

3. Sketch a graph of the following polynomial:

$$(1 + x)(2 - x)(3 + x)(1 - x) = (x)f.$$
Section 9: One Variable Statistics

The following Mathematics Florida Standards will be covered in this section:

MAFS.912.S-ID.1.1
Represent data with plots on the real number line (dot plots, histograms, and box plots).

MAFS.912.S-ID.1.2
Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

MAFS.912.S-ID.1.3
Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Topics in this Section

Topic 1: Dot Plots

Topic 2: Histograms

Topic 3: Box Plots – Part 1

Topic 4: Box Plots – Part 2

Topic 5: Measures of Center and Shapes of Distribution

Topic 6: Measuring Spread – Part 1

Topic 7: Measuring Spread – Part 2

Topic 8: The Empirical Rule

Topic 9: Outliers in Data Sets

Statistics is the science of collecting, organizing, and analyzing data.

There are two types of quantitative data:

- Discrete: There is a finite number of possible data values.
- Continuous: There are too many possible data values to count.

Two major classifications of data:

- Categorical: Data are based on "qualities" such as color, taste, or texture, rather than measurements
- Quantitative: Data are based on measurements of data values.

MAFS.912.S-ID.1.1
Represent data with plots on the real number line (dot plots, histograms, and box plots).

MAFS.912.S-ID.1.2
Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
Section 9: One Variable Statistics

- Continuous quantitative
- Discrete quantitative
- Categorical

**Number of Children**
- Continuous quantitative
- Discrete quantitative
- Categorical

**Distance a Football is Thrown**
- Continuous quantitative
- Discrete quantitative
- Categorical

**Area of a Square**
- Continuous quantitative
- Discrete quantitative
- Categorical

**Household Income**
- Continuous quantitative
- Discrete quantitative
- Categorical

**Favorite Subject**
- Continuous quantitative
- Discrete quantitative
- Categorical

**Height**
- Continuous quantitative
- Discrete quantitative
- Categorical

**Classify the following variables:**

- Quantitative
  - The average is meaningful when the data is quantitative. The average of height data is meaningful.

- Categorical
  - To differentiate between quantitative and categorical data ask yourself: Can I take the average of this data and is it meaningful?
Section 9: One Variable Statistics

b. What observations can you make about the shape of the distribution?

D. Construct a dot plot of the data above.

1. The amount of time 26 students spent on their phone on a given day (rounded to the nearest hour) is recorded as follows:

0. 3, 4, 5, 6, 7, 7, 7, 7, 8, 8, 9, 9, 9, 9, 10, 10, 10, 11, 11, 12, 12

2. The responses below are from Mrs. Fernando's class survey:

- 0, 2, 3, 4, 8, 10, 12, 13, 14

2. Are there any values that seem not to fit? Justify your answer.

C. What does the distribution tell you about the number of dollars each student has?
Consider the following sample displayed using a histogram.

1. 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 5, 5, 5, 6, 6, 7

Describe the data from the dot plot.

1. The cafeteria at Just Dance Academy offers items of

2. Beat the Test!
Section 9: One Variable Statistics

A histogram is a bar-style data display showing frequency of data measured over _________________, rather than displaying each individual data value.

1. Construct a histogram to represent the data:

0.2, 2.2, 3.3, 3.3, 3.3, 3.3, 4.4, 4.4, 5.5, 5.5, 6.6, 6.6

Describe an Interval.

- Larger sets of data
- Continuous data

Histograms are often used for:

- Describe the graph and both axes.
- Always the graph and the y-axis.
- Choose the appropriate intervals on the x-axis.
- Each interval length must be the same.

Never exceed data displayed over.

A histogram is a bar-style data display showing frequency of data measured over.

Let's Practice!
Determine the sets of data where it would be better to use a histogram than a dot plot.

1. Average daily weather temperatures for Orlando over a month
2. Daily weather temperatures for Orlando over a year
3. The results of rolling two dice over and over
4. The time, rounded to the nearest second, needed to run a 100-meter race for 125 randomly selected athletes
5. Height of high school football players statewide
6. The local men’s basketball team had a great season. The total points scored by the team for each of the 20 games are listed below:

46, 46, 46, 46, 52, 53, 53, 55, 55, 56, 57, 57, 57, 58, 62, 62, 62, 64, 64, 65, 65, 67, 67, 67, 67, 68, 68, 69

Create a frequency table and construct a histogram of the data.
Section 9: One Variable Statistics

What is a quartile?

Odd data set

Even data set

The five-number summary for a data set consists of

minimum, first quartile, median, third quartile, and maximum values.

A box plot displays the five-number summary for a data set.

The following box plot graphically displays a summary of the data set (1, 2, 3, 3, 3.3, 4, 4, 4, 4, 5, 5, 5, 6, 6.7).

Box Plots - Part I

Section 7 - Topic 3
Section 9: One Variable Statistics

Some observations from our boxplot:

- The lowest 25% of data values are from ____ to ____.
- The highest 75% of data values are from ____ to ____.
- The middle (the box area) represents the values from ______ to ______.
- The middle is also known as the IQR (interquartile range).
- The third quartile represents the first 75% of the data.
- The first quartile represents the lower 25% of the data.
- The middle 50% is also known as the IQR.
- The middle 50% (the box area) represents the values from ______ to ______.
- The highest 25% of data values are from ______ to ______.
- The lowest 50% of data values are from ______ to ______.

Consider the following data set with an even number of data values:

6.2, 7.4, 7.3, 8.5
Section 9: One Variable Statistics

Compare and contrast both box plots. Explain which boxplot is not symmetrical. Justify your answer.

**Let's Practice!**

1. Consider the following data set with an odd number of data values:

| Data Values: | 3.7, 11.1, 15, 18.2 |

   a. The minimum value of the data set is ______.
   b. The maximum value of the data set is ______.
   c. The median of the data set is ______.
   d. The first quartile of the data set is ______.
   e. The third quartile of the data set is ______.

**Other.** Construct the box plots for both data sets. One above the other.

| Data Set #1: | 1.3, 5.7, 9.9, 11.1, 13.2 |
| Data Set #2: | 1.3, 3, 5.7, 9.9, 11.1, 13.2 |

Consider the following data sets:

**Box Plots - Part 2**

Section 9 - Topic 4
Section 9: One Variable Statistics

Part A: Use the following vocabulary to label the box plot.

A. Average
B. First Quartile
C. Maximum
D. Mean
E. Median
F. Minimum
G. Third Quartile

Hint: You will not use all of the words on the list.

Part B: The 50th percentile of the data set is ________.

Part C: Construct a box plot to represent the data. Label the
minimum, maximum, first quartile, third quartile, and median.

Day was recorded as follows (cal Island is not actually on
a given island).

1. Mrs. Bridgewater recorded the number of Snapchats 10
sent in one day and constructed the
different students sent in one day and constructed the

Try It

1. Use the five-number summary to construct a box plot.

b. The bottom 25% of tourists spent, at most, ________
hours.
Section 9: One Variable Statistics

Measures of Center and Shapes of Distributions

Data displays can be used to describe the following elements of a data set's distribution:

- **Center**
- **Shape**
- **Spread**

There are three common measures of center:

- **Mean** – The ______________ of the data values.
- **Median** – The ______________ value of the ordered data set.
- **Mode** – The ______________ occuring value(s) of the data values.

Data.

Part E: Add dots to the number line below to complete the dot plot so that it could also represent the data.

Per day:
- 10
- 12
- 13
- 14
- 15

Part D: 75% of students send 12 or fewer snapshots per day.

Part C: Half of the data values are between 2 and 20.
Section 9: One Variable Statistics

### Practice

<table>
<thead>
<tr>
<th>Score</th>
<th>60</th>
<th>65</th>
<th>70</th>
<th>75</th>
<th>80</th>
<th>85</th>
<th>90</th>
<th>95</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
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<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The dot plot for the data is as follows:

60, 60, 70, 70, 70, 70, 70, 80, 80, 80, 90, 90, 90, 90, 90, 90, 90, 90, 100, 100

### Questions

1. Why not? 
   - The shape of the distribution is not symmetric, and follows a normal distribution. Why do you notice about the shape of a normal distribution follows a normal distribution. The dot in the previous dot plot is

2. Which measure is a more appropriate measure of center? 
   - The mean or the median?

3. What are the mean and the median of this data set?

Looking at the dot plot, what do you think is the value of the median?

Looking at the dot plot, what do you think is the value of the mean?

### Activities

Mr. Gray gave a test on a regular school day with no special activities. The scores are listed below:

60, 60, 70, 70, 70, 70, 70, 80, 80, 80, 90, 90, 90, 90, 90, 90, 90, 90, 100, 100
Section 9: One Variable Statistics

1. Mr. Logan surveyed his junior and senior students about the time they spent studying math in one day. He then tabulated the results and created a dot plot displaying the data for both groups.

Part A: Using one to two sentences, describe the center and shapes for the groups. The difference between the number of minutes the juniors and seniors studied by comparing the dot plots is ______.

Part B: The value of the larger mean for the two groups is ______.

Part C: The value of the larger median for the two groups is ______.

The dot plots for the data on minutes spent studying show that the mean is higher for seniors, whereas the median is higher for juniors.

Try It

a. Which value do you think will be smaller, the mean or the median of the students' scores? Explain.

50, 60, 70, 80, 80, 90, 90, 90, 90, 90, 100, 100

b. The score distribution was as follows:

Early release day, the scores were as follows:

Mr. Gray then gave a test the day after a mid-week
Section 9: One Variable Statistics

What are the differences between the spreads of the two data sets?

There are two primary ways to measure the spread of data:

- **Interquartile Range (IQR)**
- **Standard Deviation**

The IQR is typically used to describe the spread of skewed data.

Consider the following data set:

5, 5.6, 7.8, 8.9, 10, 12.12

What are the first and third quartiles of the data?

\[ \text{Interquartile Range (IQR)} = \]

There are two primary ways to measure the spread of data:

- **IQR**
- **Standard Deviation**

Why do you think IQR is used to measure spread in skewed data?

Measure of spread tells us how much a data sample is spread out or scattered.

A meteorologist in a different state also recorded the average temperature weekly temperatures over a 13-week period and displayed the data below.

A meteorologist recorded the average weekly weather temperatures over a 13-week period and displayed the data below.

1. Measures of Spread - Part 1
The data for the Bozeman blocks is skewed to the left.

- The data for the Aggie times has a longer right tail than the left.
- The interquartile range of the Bozeman blocks is less than the interquartile range of the Aggie times.
- The median time of the Bozeman blocks is less than the median time of the Aggie times.
- The highest 25% of Aggie times on both teams complete the obstacle course in about the same amount of time.

Bozeman blocks and the Aggie times:

- Which statements are true about the data for the
Bozeman blocks and the Aggie times? Select all that apply.

Which has a larger standard deviation? Explain your answer.

Consider the dot plots below:

<table>
<thead>
<tr>
<th>1.3</th>
<th>2.4</th>
<th>3.5</th>
<th>4.6</th>
<th>5.7</th>
<th>6.8</th>
<th>7.9</th>
<th>8.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.0</td>
<td>6.8</td>
<td>7.4</td>
<td>7.6</td>
<td>8.0</td>
<td>8.2</td>
<td>8.5</td>
<td>9.0</td>
</tr>
<tr>
<td>5.2</td>
<td>5.8</td>
<td>6.2</td>
<td>6.3</td>
<td>6.7</td>
<td>7.0</td>
<td>7.3</td>
<td>8.0</td>
</tr>
<tr>
<td>4.3</td>
<td>5.0</td>
<td>5.2</td>
<td>5.4</td>
<td>6.0</td>
<td>6.2</td>
<td>7.0</td>
<td>7.5</td>
</tr>
<tr>
<td>2.1</td>
<td>3.1</td>
<td>3.2</td>
<td>3.3</td>
<td>4.0</td>
<td>4.0</td>
<td>4.3</td>
<td>4.9</td>
</tr>
</tbody>
</table>

Let's Practice!

Measures of spread: Part 2

Section 9 - Topic 7

Standard deviation is the typical distance of the data from the mean.

Typically used for the individual values are

- Typically used for the individual values are
- Typically used for the individual values are
Section 9: One Variable Statistics

Data on the time that Mrs. Lanister's students spend studying math and science on a given night is summarized below:

<table>
<thead>
<tr>
<th>Subject</th>
<th>Mean</th>
<th>Minimum</th>
<th>First Quartile</th>
<th>Median</th>
<th>Third Quartile</th>
<th>Maximum</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Science</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tyrion spent 60 minutes studying math and 50 minutes studying science. If Tyrion spent 72 minutes studying math, which of the following would be affected? Check all that apply.

- Increase
- Decrease
- Stays the Same
- Interquartile Range for Math Time
- Standard Deviation of Math Time

Try the Test

1. Data on the time that Mrs. Lanister's students spend studying math and science on a given night is summarized below:

<table>
<thead>
<tr>
<th>Subject</th>
<th>Mean</th>
<th>Minimum</th>
<th>First Quartile</th>
<th>Median</th>
<th>Third Quartile</th>
<th>Maximum</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Science</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   a. Which major has the largest median salary?
      - Psychology
      - Engineering
      - Business

   b. Which major has the largest interquartile range?
Section 9: One Variable Statistics

The Empirical Rule

Section 9 - Topic 8

Assume that we have a data set so large that we are not given a list of all the values. We are told the data follows a normal distribution with a mean of \( \mu \) and standard deviation of \( \sigma \).

Label the distribution below with the values using the mean and standard deviation.

Suppose an observation is 1.5 standard deviations above the mean.

Suppose one of the data values is 8. An observation of 8 is ______ standard deviation(s) _______ the mean.

Suppose an observation is 2 standard deviations below the mean.

Suppose one of the data values is 20. An observation of 20 is ______ standard deviation(s) _______ the mean.

Suppose an observation is ______ standard deviations above the mean. The value of that observation is _________.

We can use the empirical rule to understand the data distribution.

The appropriate measure of spread is the interquartile range. Part A: The boxplot below represents the data. Calculate the quartiles. The boxplot is skewed to the right.

2. The data from a survey of the ages of people in a CrossFit class was skewed to the right.
Section 9: One Variable Statistics

Let's Practice!

1. Suppose the amount of water a machine dispenses into plastic bottles has a normal distribution with a mean of 16.0 ounces and a standard deviation of 0.1 ounces.

   a. What percentage of bottles contain between 15.8 and 16.2 ounces of water?

   b. What percentage of bottles contain between 16.4 and 16.6 ounces of water?

   c. Approximately 68% of bottles have between 15.9 and 16.1 ounces of water. The middle 95% of bottles contain between 15.8 and 16.2 ounces of water.

   d. What is the probability that a randomly selected bottle contains less than 15.9 ounces of water?

   e. What percentage of bottles contain more than 16.4 ounces of water?

   f. Suppose the amount of water a machine dispenses into plastic bottles has a normal distribution with a mean of 16.0 ounces and a standard deviation of 0.1 ounces. What percentage of bottles contain more than 16.3 ounces of water?

   g. What is the probability that a randomly selected bottle contains less than 15.9 ounces of water?

   h. What percentage of bottles contain between 15.8 and 16.2 ounces of water?
Section 9: One Variable Statistics

1. SAT mathematics scores for a particular year are approximately normally distributed with a mean of and a standard deviation of 80.

Part A: A student who scores a 750 is in the ___________ percentile.

Part B: What is the probability that a randomly selected score is greater than 670?

Part C: What is the probability that a randomly selected score is greater than 670?

Part D: What percentage of students score between 350 and 670?
Grandma Gadget’s data point is called an outlier.

A **_outlier_** is an ___________________ value in a data set that is very distant from the others.

**Let’s Practice!**

1. The number of customers received by a car dealership during 30 randomly selected days is listed:

<table>
<thead>
<tr>
<th>33</th>
<th>37</th>
<th>29</th>
<th>37</th>
<th>35</th>
<th>29</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>37</td>
<td>26</td>
<td>31</td>
<td>37</td>
<td>33</td>
</tr>
<tr>
<td>35</td>
<td>35</td>
<td>28</td>
<td>34</td>
<td>34</td>
<td>34</td>
</tr>
<tr>
<td>32</td>
<td>35</td>
<td>31</td>
<td>26</td>
<td>31</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>29</td>
<td>33</td>
<td>27</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   Identify the outlier, and describe how it affects the mean and the standard deviation.

   The outlier is _______. The outlier in the data set causes the mean to _________________ and the standard deviation to ___________________.

2. A survey about the average number of text messages sent per day was conducted at a retirement home:

   - The mean for this data set is 8.7 and the median is 10.
   - 5, 5, 5, 5, 10, 10, 10, 10, 10, 10.

   Grandma Gadget is up-to-date on the latest technology and loves to text her grandchildren. Her data is substituted for one of the original values of 15. She sends an average of 85 texts per day.

   Does Grandma Gadget’s data have a greater effect on standard deviation or interquartile range? Justify your answer.

   **Answer:**

   Grandma Gadget – the mean or the median? Justify your answer.

   Which measure of center will be most affected by substituting Grandma Gadget’s data?

   **The new data set is:**

   5, 5, 5, 5, 10, 10, 10, 10, 10, 10.

   The outlier is in the data set.

   An outlier is a data point that is very distant from the others.

   **Outliers in Data Sets**

   **Section 9 – Topic 9**
The students in Mrs. Gomez's class were surveyed about the number of text messages they send per day. The data is as follows:

a. What value would you predict to be an outlier?

b. How does the outlier affect the mean?

c. How does the outlier affect the median?

d. Which measure of center would best describe the data, the mean or the median?

e. How does the outlier affect the standard deviation?

f. How does the outlier affect the interquartile range?

g. Which measure of spread would best describe the data, the standard deviation or the interquartile range?
Section 9: One Variable Statistics

- Stay the same
- Decrease
- Increase

Standard deviation would

- Stay the same
- Decrease
- Increase

Range would

- Greater than the
- Mean
- Median

- The outlier causes the
- Mean
- Median to be

Part C: If the outlier was changed to 5, the interquartile

Part A: Are there any outliers in the data set above?

4.3, 19, 1.2, 2.4, 3.5, 3.4, 5.4, 5

The values are as follows:

2. After a long day at Disney World, a group of students were asked how many times they rode Space Mountain.
## Section 10: Two Variable Statistics

The following Mathematics Florida Standards will be covered in this section:

<table>
<thead>
<tr>
<th>MAFS.912.S-ID.2.5</th>
<th>Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAFS.912.S-ID.2.6</td>
<td>Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. Formally assess the fit of a function by plotting and analyzing residuals.</td>
</tr>
<tr>
<td>MAFS.912.S-ID.3.7</td>
<td>Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.</td>
</tr>
<tr>
<td>MAFS.912.S-ID.3.8</td>
<td>Compute (using technology) and interpret the correlation coefficient. Interpret the correlation coefficient and determine if it makes sense in the context of the data.</td>
</tr>
<tr>
<td>MAFS.912.S-ID.3.9</td>
<td>Distinguish between correlation and causation.</td>
</tr>
</tbody>
</table>
Section 10: Two Variable Statistics

### Relationship between Two Categorical Variables

- **Marginal and Joint Probabilities**

A two-way frequency table is a visual representation of the frequency counts for each categorical variable. The table can also be called a contingency table.

<table>
<thead>
<tr>
<th></th>
<th>Math</th>
<th>English</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Females</strong></td>
<td>35</td>
<td>25</td>
<td>60</td>
</tr>
<tr>
<td><strong>Males</strong></td>
<td>42</td>
<td>47</td>
<td>89</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>77</td>
<td>72</td>
<td>149</td>
</tr>
</tbody>
</table>

The total frequency for any row or column is called a **marginal frequency**.

Why do you think these total frequencies are called marginal frequencies?

Joint frequencies are the counts in the body of the table.

Why do you think these frequencies are called joint frequencies?

Two categorical variables can be represented with a two-way frequency table.

### Consider the following survey:

There were 149 elementary students interviewed and asked to choose whether they prefer math or English class. The data was broken down by gender:

- 25 females prefer English class
- 35 females prefer math class
- 47 males prefer English class
- 42 males prefer math class

Consider the following survey:
The frequency table can be easily changed to show relative frequencies.

To calculate relative frequency, divide each count in the frequency table by the overall total.

Complete the following relative frequency table.

**Elementary Students Survey**

<table>
<thead>
<tr>
<th>Math</th>
<th>English</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Males</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Females</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Why do you think these ratios are called relative frequencies?

Draw a box around the marginal relative frequencies and circle the joint relative frequencies in the table.

Interpret the marginal relative frequency for female students.

Interpret the joint relative frequency for male students who prefer math.

**Section 10 – Topic 2**

Let’s Practice!

1. A survey of high school students asked if they play video games. The following frequency table was created based on their responses.

<table>
<thead>
<tr>
<th>Plays Video Games</th>
<th>Does Not Play Video Games</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Males</td>
<td>9</td>
<td>65</td>
</tr>
<tr>
<td>Females</td>
<td>5</td>
<td>69</td>
</tr>
<tr>
<td>Total</td>
<td>14</td>
<td>134</td>
</tr>
</tbody>
</table>

a. Compute the joint and marginal relative frequencies in the table.

b. How many female students do not play video games?

c. What percentage of students interviewed were females who do not play video games?

**Video Games Survey**

on their responses.

Let’s Practice!

Marginal and Joint Probabilities – Part 2

**Relationship between Two Categorical Variables**

Section 10 – Topic 2

To calculate relative frequencies, divide each count in the frequency table by the overall total.
### Part C: What does the relative frequency represent?

- a. Consider the frequency table Video Games Survey.
- b. How many male students were interviewed?
- c. What is the probability that a student interviewed is male?
- d. Which numbers represent joint frequencies?
- e. Which numbers represent joint relative frequencies?
- f. What percentage of the subjects interviewed play video games?

### Part D: Circle the smallest marginal frequency.

<table>
<thead>
<tr>
<th>Handed</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right- handed</td>
<td>?</td>
</tr>
<tr>
<td>Left- handed</td>
<td>?</td>
</tr>
<tr>
<td>Total</td>
<td>?</td>
</tr>
</tbody>
</table>

### Part B: What is the joint relative frequency for right-handed freshmen?

- ?

### Part C: What does the relative frequency represent?

- ?

### Part D: Circle the smallest marginal frequency.

- ?

---

**Try It!**

1. A survey conducted at Ambidextrous High School asked students to indicate their grade level and whether they were left-handed or right-handed. Only 9% of the 491 freshmen are left-handed. Out of the 379 students in the sophomore class, 289 of them are right-handed. There are 433 students in the junior class and 12 of them are left-handed. There are 37 right-handed seniors.

Part A: Complete the frequency table to display the results of the survey.

<table>
<thead>
<tr>
<th>Handed</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right- handed</td>
<td>?</td>
</tr>
<tr>
<td>Left- handed</td>
<td>?</td>
</tr>
<tr>
<td>Total</td>
<td>?</td>
</tr>
</tbody>
</table>

Part B: What is the joint relative frequency for right-handed freshmen?

- ?

Part C: What does the relative frequency represent?

- ?

Part D: Circle the smallest marginal frequency.

- ?
Section 10: Two Categorical Variables

Section 10 – Topic 3

Relationship between Two Categorical Variables

Conditional Probabilities

Recall the students’ class preference data:

<table>
<thead>
<tr>
<th></th>
<th>Math</th>
<th>English</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Males</td>
<td>49</td>
<td>42</td>
<td>91</td>
</tr>
<tr>
<td>Females</td>
<td>72</td>
<td>35</td>
<td>107</td>
</tr>
<tr>
<td>Total</td>
<td>149</td>
<td>77</td>
<td>226</td>
</tr>
</tbody>
</table>

The principal says that males in the interview have a stronger preference for math than females. Why might the principal say this?

We can determine the answer to questions like this by comparing conditional relative frequencies.

Complete the following table to determine whether males or females showed stronger math preference in the survey.

Conditional Relative Frequency Table

<table>
<thead>
<tr>
<th></th>
<th>Math</th>
<th>English</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Males</td>
<td>49/91</td>
<td>42/91</td>
<td>91</td>
</tr>
<tr>
<td>Females</td>
<td>72/107</td>
<td>35/107</td>
<td>107</td>
</tr>
<tr>
<td>Total</td>
<td>149/226</td>
<td>77/226</td>
<td>226</td>
</tr>
</tbody>
</table>

What percentage of male students prefer Math?

What percentage of female students prefer Math?

These percentages are called conditional relative frequencies.

Make a conjecture as to why they are called conditional relative frequencies.

When trying to predict a person’s class preference, does it help to know their gender?
When we evaluate whether there seems to be a relationship between two categorical variables, we look at the conditional relative frequencies. If there is a significant difference between the conditional relative frequencies, then there is evidence of an association between two categorical variables.

Is there an association between gender and class preference?

Let's Practice!

Consider the high school students who were asked if they play video games:

<table>
<thead>
<tr>
<th>Video Games Survey</th>
<th>Does Not Play</th>
<th>Plays</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Males</td>
<td>129</td>
<td>9</td>
<td>138</td>
</tr>
<tr>
<td>Females</td>
<td>5</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Total</td>
<td>134</td>
<td>145</td>
<td>279</td>
</tr>
</tbody>
</table>

1. What percentage of the students who do not play video games are female?

2. Given that a student is female, what is the probability that the student does not play video games?

3. Of the students who are male, what is the probability that the student plays video games?

4. What percentage of the students who play video games are male?
Freshmen and sophomores were asked about their preferences for an end-of-year field trip for students who pass their final examinations. Students were given the choice to visit an amusement park, a water park, or a mystery destination. A random sample of freshmen and sophomores was selected. The activities coordinator constructed a frequency table to analyze the data:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Freshmen</th>
<th>Sophomores</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amusement Park</td>
<td>25</td>
<td>35</td>
<td>60</td>
</tr>
<tr>
<td>Water Park</td>
<td>10</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>Mystery Destination</td>
<td>15</td>
<td>25</td>
<td>40</td>
</tr>
<tr>
<td>Total</td>
<td>55</td>
<td>55</td>
<td>110</td>
</tr>
</tbody>
</table>

**Part A:** What does the relative frequency represent?
**Part B:** What percentage of students who want to go to an amusement park are sophomores?
**Part C:** What activity should the coordinator schedule for sophomores? Justify your answer.

### Scatter Plots and Function Models

Consider the data below of the statewide average temperature in Florida each year since 1999. A scatterplot of the data is also shown on the following page.

<table>
<thead>
<tr>
<th>Year</th>
<th>Average Statewide Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999</td>
<td>71.2</td>
</tr>
<tr>
<td>2000</td>
<td>71.6</td>
</tr>
<tr>
<td>2001</td>
<td>71.8</td>
</tr>
<tr>
<td>2002</td>
<td>71.0</td>
</tr>
<tr>
<td>2003</td>
<td>70.7</td>
</tr>
<tr>
<td>2004</td>
<td>70.8</td>
</tr>
<tr>
<td>2005</td>
<td>70.3</td>
</tr>
<tr>
<td>2006</td>
<td>70.5</td>
</tr>
<tr>
<td>2007</td>
<td>70.6</td>
</tr>
<tr>
<td>2008</td>
<td>70.9</td>
</tr>
<tr>
<td>2009</td>
<td>71.0</td>
</tr>
<tr>
<td>2010</td>
<td>71.4</td>
</tr>
<tr>
<td>2011</td>
<td>71.1</td>
</tr>
<tr>
<td>2012</td>
<td>70.9</td>
</tr>
<tr>
<td>2013</td>
<td>71.2</td>
</tr>
</tbody>
</table>

Consider the data below involving two variables:

<table>
<thead>
<tr>
<th>Year (1999 – 2013)</th>
<th>Average Temperature (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999</td>
<td>71.2</td>
</tr>
<tr>
<td>2000</td>
<td>71.6</td>
</tr>
<tr>
<td>2001</td>
<td>71.8</td>
</tr>
<tr>
<td>2002</td>
<td>71.0</td>
</tr>
<tr>
<td>2003</td>
<td>70.7</td>
</tr>
<tr>
<td>2004</td>
<td>70.8</td>
</tr>
<tr>
<td>2005</td>
<td>70.3</td>
</tr>
<tr>
<td>2006</td>
<td>70.5</td>
</tr>
<tr>
<td>2007</td>
<td>70.6</td>
</tr>
<tr>
<td>2008</td>
<td>70.9</td>
</tr>
<tr>
<td>2009</td>
<td>71.0</td>
</tr>
<tr>
<td>2010</td>
<td>71.4</td>
</tr>
<tr>
<td>2011</td>
<td>71.1</td>
</tr>
<tr>
<td>2012</td>
<td>70.9</td>
</tr>
<tr>
<td>2013</td>
<td>71.2</td>
</tr>
</tbody>
</table>

Let's consider quantitative data involving two variables.
Section 10.2: Two Variable Statistics

Let's Practice!

1. A scatterplot is a graphical representation of the relationship between two quantitative variables. Classify the relationship represented in each of the scatterplots below as linear, quadratic, or exponential.

2. Describe the relationship between year and temperature using the line of best fit.

What does the ordered pair (3, 7.4) represent?

What do the values on the y-axis represent?

What do the values on the x-axis represent?

Average Flood Temperature for Years 1999 – 2013
a. What is the predicted number of posts for month 11 using the linear function?

b. What is the predicted number of posts for month 11 using the exponential function?

c. Is the linear equation or the exponential equation the best model for this data?
Let's consider which function should be used to fit the data - the linear function $f(x) = 176.32x - 380.47$ or the exponential function $g(x) = 23.30 - 1.62x$.

If the trend continues, predict the number of mandeels deaths that will be killed by watercraft in 2020.

The linear equation that best models this relationship is $y = 4.3697x + 31.667$, where $x$ represents the number of years since 1990 and $y$ represents the number of mandeels.

The scatterplot below shows the number of mandeels killed by watercraft each year in Florida from 1991 – 2000.
A residual is the vertical distance between an actual data point and the function fitted to the data.

Residual = actual – predicted

### Compute the residuals for each function:

#### Linear Function:

<table>
<thead>
<tr>
<th>Month</th>
<th>Posts</th>
<th>Predicted Value</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>24</td>
<td>-15</td>
</tr>
<tr>
<td>2</td>
<td>52</td>
<td>27</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>49</td>
<td>49</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td>14</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>34</td>
<td>34</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>324</td>
<td>324</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>1529</td>
<td>1529</td>
<td>0</td>
</tr>
</tbody>
</table>

#### Exponential Function: \( b(x) = 23.30 \times 1.62^x \)

<table>
<thead>
<tr>
<th>Month</th>
<th>Posts</th>
<th>Predicted Value</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>75</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>52</td>
<td>15</td>
<td>37</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>91</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td>14</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>34</td>
<td>421</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>324</td>
<td>142</td>
<td>9</td>
</tr>
<tr>
<td>7</td>
<td>1529</td>
<td>91</td>
<td>111</td>
</tr>
</tbody>
</table>

### Residual Plot

A residual plot is a graph of the residuals (y-axis) versus the x-values (x-axis).

Residual Plot: For the linear function \( f(x) = 176.32x - 380.47 \):

To determine whether or not a function is a good fit, look at a residual plot of the data.

### Two models: What do you notice about the values of the residuals for the point and the function fitted to the data?

A residual is the vertical distance between an actual data point and the function fitted to the data.
Consider the residual plots for the linear and exponential models. Which function fits the data better – the linear or the exponential function? How do you know?
Let's Practice!

1. If a data set has a quadratic trend and a linear function is fit to the data, what will the residual plot look like?

2. If a data set has a quadratic trend and a linear function is fit to the data, what will the residual plot look like?
A test for the data was an appropriate fit for the data, as shown below. Circle the plots that indicate that the linear regression residual plots for each data set are linear relationships. Suppose a quadratic function is fit to a set of data, which
The correlation coefficient, \( r \), measures the strength and direction of the linear association between two quantitative variables.

The closer \( r \) is to 0, the weaker the relationship. 

The absolute value of \( r \) will be.

\( r = +0.450 \) and \( r = -0.450 \) both indicate the strength of association between the variables.

\( r = -0.560 \) and \( r = 0.560 \) both indicate the strength of association between the variables.

\( r = 0 \) indicates no linear association.

Describe the relationship between the years since 1990 and the number of Manatees killed by underwater in Florida.

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of Deaths</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>100.0</td>
</tr>
<tr>
<td>1991</td>
<td>95.0</td>
</tr>
<tr>
<td>1992</td>
<td>90.0</td>
</tr>
<tr>
<td>1993</td>
<td>85.0</td>
</tr>
<tr>
<td>1994</td>
<td>80.0</td>
</tr>
</tbody>
</table>

The scatter plot below shows the number of Manatees killed by underwater in each year from 1991 to 2000.
Correlation coefficient means:

For the data above, \( r = -0.639 \). What does the value of the

1. **Let's Practice!**

The table and scatter plot below show the relationship

<table>
<thead>
<tr>
<th>Student</th>
<th>Final Grade</th>
<th>Missed Classes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>88</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>95</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>93</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>92</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>98</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>96</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>98</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>99</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>98</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>96</td>
<td>1</td>
</tr>
</tbody>
</table>

2. **For the data below, \( r = +0.828 \). What does the value of the

3. Try it!
There is a strong positive association between the amount of fire damage (\(y\)) and the number of firefighters on the scene (\(x\)). Does having more firefighters on the scene cause greater fire damage?

Correlation does not imply causation! Causation is when one event causes another to happen. Two variables can be correlated without one event causing the other. Two variables can be correlated without one event causing the other.

1. Determine whether the correlation in each situation below implies causation, select all that apply.

- A person’s height is positively correlated to his/her weight.
- A person’s height is positively correlated to his/her weight.
- The amount of gas left in the gas tank.
- The number of miles driven is negatively correlated to the daily ice cream sales in Florida.
- The number of sharks attacks.
- Cigarettes and lung cancer.
- There is a positive correlation between smoking cigarettes and lung cancer.
- There is a positive correlation between smoking cigarettes and lung cancer.

2. Correlation does not imply causation! Causation is when one event causes another to happen. Two variables can be correlated without one event causing the other.

- There is a strong positive association between the amount of fire damage and the number of firefighters on the scene.